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COLLINEAR RADIATIVE ELECTRON-POSITRON SCATTERING IN LEADING LOGARITHMIC APPROXIMATION

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Radiative corrections to the cross section of radiative large-angle Bhabha scattering process at high energies are calculated. We consider the kinematics in which a hard photon is emitted along one of momentum directions of charged fermions. Contributions coming from the emission of virtual, soft and hard additional photons are taken into account in the leading logarithmical approximation.

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The problem, this paper is dealt with, is mainly motivated by the experimental needs of measuring the cross section of the large-angle electron-positron scattering process to a per mille level accuracy, as this process is used for precise determination of the main characteristics of colliding beams – the luminosity.

To reach a one per mille accuracy one should evaluate the radiative corrections (RC) up to third order in the leading logarithmic approximation (LLA) and up to second order in the next-to-leading approximation. In a series of papers definite sources of these corrections were considered in detail [1 – 4].

In a recent publication [4] the contribution due to virtual and soft photon corrections to large–angle radiative Bhabha scattering was calculated in the kinematics in which a hard photon is emitted at large angle with respect to all charged particles momenta. In the present letter we consider the particular kinematics in which the photon moves within a narrow cone of small opening angle $\theta_0 \ll 1$ together with one of the incoming or outgoing charged particles.

In the experimental set-up with detecting of scattered electron and positron one cannot distinguish events with electron alone and those with electron accompanied by a hard photon moving at small angle $\theta < \theta_0 \ll 1$ with respect to the direction of motion of the electron. When the photon is emitted off initial particles the back-to-back kinematics[2)] will be violated, whereas in the case of its emission along the scattered particles this kinematics does hold. The quantity θ_0 in the case of emission along the final particles may be associated with the aperture of the detectors.

Upon integration over the photon angular variables, the cross section of the process

$$e(p_1) + \bar{e}(p_2) \to e(p_1') + \bar{e}(p_2') + \gamma(k_1) \tag{1}$$

in the lowest order of perturbation theory takes the form

$$\left(\frac{d\sigma_0^\gamma}{dxdc}\right)_A = \frac{4\alpha^3}{s}\left[\frac{1+(1-x)^2}{x}L_0 - 2\frac{1-x}{x}\right] \times \tag{2}$$

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2) Hereafter the center-of-mass (CMS) reference frame of initial particles is assumed.

$$\times \left(\frac{3-3x+x^2+2cx(2-x)+c^2(1-x+x^2)}{(1-x)(1-c)a^2}\right)^2 (1+\mathcal{O}(\theta_0^2)),$$

$$L_0 = 2\ln\frac{\varepsilon\theta_0}{m}, \qquad s = 4\varepsilon^2,$$

where the subscript A on the left-hand side has been used to denote the kinematics in which the hard photon is emitted along the initial electron. The quantity $x = k_1^0/\varepsilon$ is the energy fraction of the hard photon, ε is the CMS energy of the electron, m is its mass, and $c = \cos\widehat{\mathbf{p}_1, \mathbf{p}_1'}$ is the cosine of the scattering angle. The energy fractions of scattered electron (y_1) and positron (y_2)

$$y_1 = \frac{p_1'^0}{\varepsilon} = 2(1-x)/a, \qquad a = 2-x+xc,$$

$$y_2 = \frac{p_2'^0}{\varepsilon} = (2-2x+x^2+cx(2-x))/a,$$

are completely determined, as well as the positron scattering angle, by the energy-momentum conservation law.

The cross section in the kinematics of B case in which the hard photon is emitted along the scattered electron reads

$$\left(\frac{d\sigma_0^\gamma}{dxdc}\right)_B = \frac{\alpha^3}{4s}\left[\frac{1+(1-x)^2}{x}L_0' - 2\frac{1-x}{x}\right]\left(\frac{3+c^2}{1-c}\right)^2 (1+\mathcal{O}(\theta_0^2)), \tag{3}$$

$$L_0' = 2\ln\frac{\varepsilon'\theta_0}{m}, \qquad \varepsilon' = \varepsilon(1-x).$$

Let us consider first the radiative correction due to one–loop Feynman diagrams (FD), which we label as virtual photon emission. Of this type there are as many as seventy two FD. To simplify the calculation we use the physical gauge (PG) for the real photon

$$\sum_\lambda e_\mu^\lambda e_\nu^{\lambda *} = \begin{cases} 0, & \text{if } \mu \text{ or } \nu = 0 \\ \delta_{\mu\nu} - n_\mu n_\nu, & \mu=\nu=1,2,3 \end{cases}, \qquad n = \frac{\mathbf{k}_1}{\omega_1}.$$

As was shown in [5], this choice proved useful in quantum chromodynamics. It, as well, fits perfectly to the case at hand. In PG each fermion emits *independently*, contrary to the Feynman gauge in which a leading contribution (i.e. containing large logarithm L_0) arises from the interference amplitudes of emission off different fermions. The contribution from interference terms in PG is of the order of θ_0^2. We shall systematically omit them throughout and this determines the accuracy of our approach to be
$1+\mathcal{O}(\theta_0^2 L)$, $L = L_s = \ln(s/m^2)$.

In this letter we deal with the RC to the process (1) in the LLA. Hence the accuracy of the result will be restricted by the quantity of the order of $1/L \sim 5\%$ for moderate high energy colliders such as Φ, J/Ψ–factories.

Using the crossing symmetry we may restrict ourselves to the consideration of only a certain subset of FD, namely those of scattering type with one (G,L-types FD) and two-photon exchange (B,P-types of FD). For instance in A kinematics ($\mathbf{k}_1 \parallel \mathbf{p}_1$) only nine FD become relevant. Thus we are allowed to write the matrix element squared and summed over spin states as follows

$$\Sigma|M|_A^2 = \Re e(1+\mathcal{Q}_1)\left[G+L+\frac{1}{s_1 t}(1+\mathcal{Q}_2)s_1 t(B+P)\right]. \tag{4}$$

Here we use the kinematical invariants

$$\begin{aligned} s &= (p_1+p_2)^2, \quad s_1=(p_1^s+p_2^s)^2, \quad t_1=(p_1-p_1^s)^2, \\ t &= (p_2-p_2^s)^2, \quad u=(p_1-p_2^s)^2, \quad u_1=(p_2-p_1^s)^2, \\ \chi_1 &= 2p_1k_1, \quad \chi_1^s=2p_1^sk_1. \end{aligned}$$

The crossing operators act as

$$Q_1F(s_1,t_1,s,t)=F(t,s,t_1,s_1), \qquad Q_2F(s,u,s_1,u_1)=F(u,s,u_1,s_1). \tag{5}$$

The quantities L, G denote the interference between the amplitudes, corresponding to the one–loop FD of fermion self-energy and vertex insertion, with single-photon exchange in the scattering channel, and the two Born level amplitudes, containing small denominator χ_1. The graphs of B, P-types describe the interference of two-photon exchange one–loop FD with uncrossed photon *legs*. The action of the operator Q_1 yields a contribution of one–loop FD of annihilation type, whereas Q_2 being applied to B-type FD with uncrossed photon legs provides the contribution of one–loop graphs with crossed photon legs.

The total expression of the virtual RC for the case B may be obtained from that of the case A using the substitution

$$\sum |M|_B^2 = Q\begin{pmatrix} p_1 \leftrightarrow -p_1^s \\ p_2 \leftrightarrow -p_2^s \end{pmatrix} \sum |M|_A^2. \tag{6}$$

Omitting details (results to a power accuracy, including next-to-leading contributions will be published elsewhere), we put here the main results of the calculations. The virtual correction in the A kinematics is found to be

$$\begin{aligned} \left(\frac{d\sigma_V^\gamma}{dxdc}\right)_A &= \left(\frac{d\sigma_0^\gamma}{dxdc}\right)_A \frac{\alpha}{\pi}\left[-4L_t\ln\frac{m}{\lambda}+L_{u_1}^2-L_t^2-L_{s_1}^2+\frac{1}{2}L_0\ln(1-x)+\frac{11}{3}L_t\right], \\ L_t &= \ln\frac{-t}{m^2}, \qquad L_{u_1}=\ln\frac{-u_1}{m^2}, \qquad L_{s_1}=\ln\frac{s_1}{m^2}, \end{aligned} \tag{7}$$

with λ the *fictitious* photon mass, introduced with the aim at regularizing the infrared singularities. The contribution of the soft photon emission process accompanying emission of the hard one reads

$$\begin{aligned} \left(\frac{d\sigma_S^\gamma}{dxdc}\right)_A = \left(\frac{d\sigma_0^\gamma}{dxdc}\right)_A \frac{\alpha}{\pi}\Big[4L_t\ln\frac{m\Delta\varepsilon}{\lambda\varepsilon}+\frac{1}{2}(L_s^2+L_{s_1}^2+L_t^2+L_{t_1}^2-L_{u_1}^2-L_u^2)- \\ -L_t\ln(y_1y_2)\Big], \end{aligned} \tag{8}$$

$$L_{t_1}=\ln\frac{-t_1}{m^2}, \qquad L_u=\ln\frac{-u}{m^2},$$

where $\Delta\varepsilon \ll \varepsilon$ is the uppermost energy of the soft photon in CMS.

The emission of two hard photons along initial electron with the total energy fraction x and simultaneously with the energies of each of them exceeding $\Delta\varepsilon$ gives the contribution

$$\begin{aligned} \left(\frac{d\sigma^{\gamma\gamma}}{dxdc}\right)_A &= \left(\frac{d\sigma_0^\gamma}{dxdc}\right)_A \frac{\alpha L}{\pi}\left[-\ln\frac{\Delta\varepsilon}{\varepsilon}-\frac{3}{4}+\frac{1}{2}\ln(1-x)+\frac{x\mathcal{P}_\Theta^{(2)}(x)}{4(1+(1-x)^2)}\right], \\ \mathcal{P}_\Theta^{(2)}(x) &= 2\frac{1+(1-x)^2}{x}\left[2\ln x-\ln(1-x)+\frac{3}{2}\right]+(2-x)\ln(1-x)-2x. \end{aligned} \tag{9}$$

The contributions of the kinematics in which the hard photon is emitted by initial electron whereas another one is emitted by a final electron or by initial (final) positron, together with the corresponding part of RC from virtual and soft photons

$$\left(\frac{d\sigma_0^\gamma}{dxdc}\right)_A \frac{3\alpha L}{\pi}\left[\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{3}{4}\right], \tag{10}$$

may be represented via electron structure function in the spirit of Drell-Yan formalism

$$\langle\frac{d\sigma_0^\gamma}{dxdc}\rangle_A = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}L_0\int dz_2dz_3dz_4 \quad \mathcal{D}(z_2)\mathcal{D}(z_3)\mathcal{D}(z_4) \tag{11}$$
$$\times \quad \frac{d\sigma_0}{dc}(p_1(1-x), z_2p_2; q_1, q_2),$$

with the non-singlet structure function $\mathcal{D}(z)$ [6]:

$$\begin{aligned}
\mathcal{D}(z) &= \delta(1-z)+\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\alpha L}{2\pi}\right)^n \mathcal{P}^{(n)},\\
\mathcal{P}^{(n)}(z) &= \lim_{\Delta\to 0}[\delta(1-z)\mathcal{P}_\Delta^{(n)}+\Theta(1-z-\Delta)P_\Theta^{(n)}(z)],\\
\mathcal{P}_\Delta^{(1)} &= 2\ln\Delta+\frac{3}{2}, \qquad \mathcal{P}_\Theta^{(1)}(z)=\frac{1+z^2}{1-z}, \ \ldots
\end{aligned}$$

The cross section of the hard subprocess $e(p_1z_1)+\bar{e}(p_2z_2)\to e(q_1)+\bar{e}(q_2)$ entering Eq. (11) has the form

$$\frac{d\sigma_0}{dc}(z_1p_1, z_2p_2; q_1, q_2) = \frac{8\pi\alpha^2}{s}\left[\frac{z_1^2+z_2^2+z_1z_2+2c(z_2^2-z_1^2)+c^2(z_1^2+z_2^2-z_1z_2)}{z_1(1-c)(z_1+z_2+c(z_2-z_1))^2}\right]^2.$$

The energies of the scattered fermions and their scattering angles are determined by the energy-momentum conservation law

$$\begin{aligned}
q_1^0 &= \varepsilon\frac{2z_1z_2}{z_1+z_2+c(z_2-z_1)}, \qquad q_1^0+q_2^0=\varepsilon(z_1+z_2),\\
c &= \cos\widehat{\mathbf{q_1},\mathbf{p_1}}, \qquad z_1\sin\widehat{\mathbf{q_1},\mathbf{p}_1} = z_2\sin\widehat{\mathbf{q_2},\mathbf{p_1}}.
\end{aligned}$$

Due to subsequent fragmentation, the energies of the detected fermions are

$$\varepsilon_1' = z_3q_1^0, \qquad \varepsilon_2' = z_4q_2^0$$

and in general do not coincide with those of the scattered fermions on the hard stage, whereas the scattering angles in LLA remain unchanged. Let us bring the final expressions for RC in LLA to the following form

$$\left(\frac{d\sigma^{\gamma(SV\gamma)}}{dxdc}\right)_A = \left(\frac{d\sigma_0^\gamma}{dxdc}\right)_A(1+\delta_A), \quad \left(\frac{d\sigma^{\gamma(SV\gamma)}}{dxdc}\right)_B = \left(\frac{d\sigma_0^\gamma}{dxdc}\right)_B(1+\delta_B), \tag{12}$$

with

$$\begin{aligned}
\delta_A &= \left[\left(\langle\frac{d\sigma_0^\gamma}{dxdc}\rangle-\frac{d\sigma_0^\gamma}{dxdc}\right)\left(\frac{d\sigma_0^\gamma}{dxdc}\right)^{-1}\right]_A + \frac{\alpha L}{\pi}\left[\frac{2}{3}-\ln(y_1y_2)+\frac{x\mathcal{P}_\Theta^{(2)}(x)}{4(1+(1-x)^2)}\right],\\
\delta_B &= \left[\left(\langle\frac{d\sigma_0^\gamma}{dxdc}\rangle-\frac{d\sigma_0^\gamma}{dxdc}\right)\left(\frac{d\sigma_0^\gamma}{dxdc}\right)^{-1}\right]_B + \frac{\alpha L}{\pi}\left[\frac{2}{3}+\frac{x\mathcal{P}_\Theta^{(2)}(x)}{4(1+(1-x)^2)}\right].
\end{aligned}$$

The quantities δ_A, δ_B are smooth functions of x, c of some per cent in magnitude.

In conclusion we note that the Born cross section is seen to factored out of the radiative one in LLA. This is in an agreement with a general hypothesis of factorization. Nonetheless, it is evident that the term $\ln(y_1 y_2)$ in δ_A emerging from a soft photon emission does modify the expected form of the second order splitting function $\mathcal{P}_{\Theta}^{(2)}$.

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