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SELF-INDUCED TRANSPARENCY SOLITON LASER

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Passive mode-locking technique with inhomogeneously broadened absorber in a regime when duration of generated pulse is less than polarization decay time is described theoretically. The possibility of generation of self-induced transparency solitons directly from the laser oscillator is shown. The new effect, instability due to spectral hole-burning, is discussed. Comparison with experiment is given.

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Usually, the term "soliton laser" implies a mode-locked laser, where pulse shaping is due to interplay between group velocity dispersion and Kerr-type nonlinearity. So, the pulse experiences shaping as a soliton of nonlinear Schrodinger equation (NLS). Here we refer to a different problem, when shaping mechanism appears as a result of *resonant* interaction between a short pulse and a two-level absorber. Conventional techniques of passive mode-locking includes either fast $\tau \gg T_1, T_2$ (with τ, T_1 and T_2 as pulse duration, and decay times of polarization and population difference, correspondingly), or slow, $T_2 \ll \tau \ll T_1$, saturable absorber [1]. If a pulse becomes shorter than dephasing time, $\tau \ll T_2$, the interaction is called as a coherent, and over a certain power threshold may lead to formation of a self-induced transparency (SIT) soliton [2]. This case of passive mode-locking with *coherent* absorber, or in other words, – *SIT soliton laser*, constitutes the subject of the Letter.

Currently, laser operation in the SIT regime attracts attention, but level of understanding of the problem seems far from complete. Thus, in Ref. [3] Nakazawa et.al. described the idea of forming a SIT soliton laser with an erbium-doped fiber (EDF) amplifier at room temperature as the gain medium and an EDF at 4.2 K as a pulse shaper by using the SIT effect. However, inspite of "ideal" conditions for SIT, $T^* \ll \tau \ll T_2$ (with $(T^*)^{-1}$ as inhomogeneous linewidth), the obtained pulse durations were longer than T_2, which means that saturable absorption rather than a coherent pulse formation was dominant in their experiment. Destructive role of Kerr nonlinearity of the fiber [4] can be viewed as a partial explanation for the instability of SIT regime. More complete study has to involve simultaneous solution of field equation together with Bloch equations for amplifier and absorber. The previous studies on intracavity SIT pulse regime Refs. [5] considered the coherent absorber as a homogeneously broadened medium, which is not an appropriate model for an EDF at 4.2 K.

In the Letter we develop a theory of a solid-state (fiber) SIT soliton laser with an inhomogeneously broadened absorber and obtain a solution in the form of a 2π-pulse. The solution turns out to be unstable in favor of cw operation for the parameters of the experimental setup in the Ref. [3]. This type of instability has its origin in the familiar hole-burning effect, and, thus, cannot be deduced from the conventional model of a homogeneously broadened absorber. On each round-trip a SIT pulse looses in the

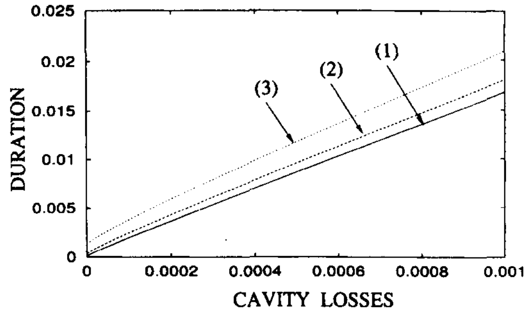

Fig.1. Normalized duration, (τ/T_{2ab}), versus the dimensionless cavity losses (l_{cav}/g_0) for different values of absorber density, (a_0/g_0): (*1*) 0.2, (*2*) 0.4, (*3*) 0.8. For parameters see Fig.2

absorber a small part of its energy ($\propto \tau/T_{2ab}$) due to incomplete Rabi flopping of the population inversion, thus transferring a certain portion of atoms, $\Delta\mathcal{N}_a(\Delta\omega)$, from the ground to the upper state. Since the population relaxes very slowly, $T_{1ab} \approx 10$ ms, compares to the cavity round-trip time $T_R \approx 660$ ns, the absorber is saturated with many ($T_{1ab}/T_R \approx 15000$) pulses (all parameters through the Letter are taken from [3]). In the steady-state the Gaussian inhomogeneous contour appears to be modified, displaying a profound dip at the center as shown in Fig.1, because the central group of atoms is saturated more heavily than others. The net absorption coefficient for the SIT pulse is the nontrivial result of two opposing tendencies: (i) the portion of the lost energy ($\propto \tau/T_{2ab}$) per unit frequency range reduces with decreasing in pulse duration; (ii) simultaneously, the additional frequency groups of atoms turn out to be involved in absorption as a result of the corresponding increasing in the pulse spectral width[1)]. On the other hand, a weak narrow-band radiation, which always presents inside a cavity due to spontaneous emission, can experience very small losses in the spectral dip burned by the circulating soliton. If absorption coefficient for the soliton appears to be larger than that for a cw radiation, the SIT operation becomes unstable. We find that the hole-burning instability takes place for relatively long SIT pulses. We also show that experimental parameters in Ref. [3] lies in the instability region, and one has to increase a pump power over a certain threshold value in order to get a stable SIT soliton operation.

The wave equation for Rabi frequency can be written in a general form

$$\frac{\partial}{\partial T}\Omega = \left(\frac{\partial}{\partial t} + \hat{O}_{bw} + \hat{O}_D + \hat{O}_g + \hat{O}_{ab}\right)\Omega\,, \tag{1}$$

where slow time variable T stands for evolution over many cavity round-trips. $\hat{O}_{bw}$ is the operator accounted for the broad-bandwidth optical filter, where we keep terms up to the second order only

$$\hat{O}_{bw} \;\rightarrow\; -l_{cav}\left[1 - (1/\Omega_f^2)(\partial^2/\partial t^2)\right]\,.$$

Nonresonant dispersive properties of the host medium is given by $\hat{O}_D$ operator, which includes GVD and SPM in the form of

$$\hat{O}_D \;\rightarrow\; i\left[D(\partial^2/\partial t^2) + K|\Omega|^2\right]\,.$$

1) The latter tendency is unique to an inhomogeneously broadened media, contrary to a homogeneously broadened medium where the absorption coefficient for a SIT soliton is always less than that for a cw radiation (apart from the effect of an intracavity spectral filter).

$\hat{O}_g$ models the broad-bandwidth homogeneously broadened amplifier[2],

$$\hat{O}_g \rightarrow g\,[1 - T_{2g}(\partial/\partial t)]\,,$$

with g obeying the following differential equation

$$\frac{d}{dT}g(T) = -\frac{g - g_0}{T_{1g}} - \frac{\Delta\mathcal{N}_g(T)}{T_R}g\,. \qquad (2)$$

where g_0 is the small signal gain, and $\Delta\mathcal{N}_g$ is the change in population difference after a single round trip,

$$\Delta\mathcal{N}_g(T) = T_{2g}\,(d_g/d_{ab})^2\;J(T)\,, \quad J(T) \equiv \int^{T_R} |\Omega|^2 dt;$$

$J(T)$ has a meaning of pulse energy[3]. Eq. (2) implies that population decay time T_{1g} is large compared to T_R, such that g responds only to the average pulse energy $J(T)$ and does not possess an appreciable time-varying component in the time scale of a single round-trip. $\hat{O}_{ab}$ describes the absorber action, which is modeled by the system of Bloch equations. The response of the absorber displays two time scales, one of which is faster than polarization decay, T_{2ab}^{-1}, and therefore corresponds to coherent pulse shaping. The other is much more slower than T_{2ab}^{-1} and T_R^{-1}, and is associated with slow response of the populations. The latter is not susceptible to the particular pulse shape and can be found as a result of averaging the Bloch variables over the cavity round-trip time. A pulse running back and forth inside the cavity selectively saturates the absorption contour transforming the initial Gaussian distribution of atoms

$$f_0(\Delta\omega) = \exp\left[-(\Delta\omega T^*)^2\right]$$

to a more complex shape, $f(\Delta\omega)$, given by

$$\frac{d}{dT}f(T,\,\Delta\omega) = -\frac{f - f_0}{T_{1ab}} - \frac{\Delta\mathcal{N}_a(T,\,\Delta\omega)}{T_R}f\,, \qquad (3)$$

where $\Delta\mathcal{N}_a(\Delta\omega)$ is the fraction of atoms with transition frequencies in the vicinity of $(\omega_0 + \Delta\omega)$ transferred to the upper state by a pulse after a single round-trip. Solving the averaged Bloch equations for the absorber, we find $\Delta\mathcal{N}_a$ for two limiting cases

$$\Delta\mathcal{N}_a^{cw} = \frac{T_{2ab}}{1 + (\Delta\omega T_{2ab})^2}\,J(T)\,, \quad \text{for} \quad \tau \gg T_{2ab}, \qquad (4)$$

$$\Delta\mathcal{N}_a = \frac{1}{T_{2ab}}\int^{T_R} |\mathcal{P}_{coh}|^2 dt\,, \quad \text{for} \quad \tau \ll T_{2ab}; \qquad (5)$$

$\mathcal{P}_{coh}$ can be found from a system of Bloch equations:

$$\partial\mathcal{P}_{coh}/\partial t = -i\Delta\omega\mathcal{P}_{coh} + \Omega\mathcal{N}\,; \qquad (6)$$

$$\partial\mathcal{N}/\partial t = -(\Omega\mathcal{P}_{coh}^* + \Omega^*\mathcal{P}_{coh})/2\,. \qquad (7)$$

[2] In the experiments [3] the homogeneous broadening dominated over inhomogeneous one for the amplifier, while for the absorber the situation was the opposite.

[3] The factor of (d_g/d_{ab}) appears as a result of normalization of Rabi frequency to the dipole moment of the absorber.

Relaxation of polarization does not contribute substantially to the pulse shaping, and the term $(-T_{2ab}\mathcal{P}_{coh})$ is dropped in the rhs of Eq. (6). However, this losses in the coherent absorber accumulate over many round-trips, and contribute to net absorption coefficient. The latter can be found in the lower limit of the perturbation theory on a small parameter τ/T_{2ab}, [2]:

$$l(T) = a_0 \frac{\langle \Delta N_a(T, \Delta\omega) \rangle}{\sqrt{\pi} T^* J(T)}\,, \tag{8}$$

where $\langle \ldots \rangle = (T^*/\sqrt{\pi}) \int \ldots f(\Delta\omega) d\Delta\omega$ is averaging over all frequency groups with the distribution function $f(\Delta\omega)$ given by a solution of Eq. (3); a_0 is an unsaturated small signal absorption of inhomogeneously broadened medium at the line center. Finally the field equation becomes

$$\frac{\partial \Omega}{\partial T} = \left\{ (g - l_{cav} - l) - \left[1 + g T_{2g}\right] \frac{\partial}{\partial t} \right\} \Omega +$$
$$+ a_0 \frac{\langle \mathcal{P}_{coh} \rangle}{\sqrt{\pi} T^*} + i \left[D \frac{\partial^2}{\partial t^2} + K |\Omega|^2 \right] \Omega + \frac{l_{cav}}{\Omega_f^2} \frac{\partial^2 \Omega}{\partial t^2}\,. \tag{9}$$

Eq. (9) together with Eqs. (2),(3), (6), and (7) describe the mode-locking operation with a coherent absorber in a self-consistent way. The system is the main result of the Letter. It contains a whole variety of different regimes arising from interesting interplay between resonant and nonresonant mode-locking mechanisms. Here, we limit ourselves by considering the fiber SIT soliton laser proposed in [3], where the effect of GVD was eliminated by shifting the zero dispersion point to the atomic resonant frequency, and the effect of optical filtering was completely negligible, $\Omega_f \tau > 100$.

Putting[4] also $K = 0$, the steady-state solution to Eqs. (2), (3), (6), and (7) takes the form of the SIT soliton

$$\Omega = 2\tau^{-1} \cosh^{-1} \left[(t - cT_R/L)/\tau \right]\,, \tag{10}$$

where L is the length of the cavity. The area of the soliton is equal to 2π resulting in the unique correspondence between duration and energy, $J = 8\tau^{-1}$. The round-trip time is given by

$$T_R = \frac{1}{c} \left[L + L_g\, g T_{2_g} + L_a \frac{a_0 \tau^2}{\sqrt{\pi} T^*} \left\langle \frac{1}{1 + (\Delta\omega\tau)^2} \right\rangle \right]\,,$$

where L_g and L_a stand for lengths of EDF amplifier and EDF absorber correspondingly. In the steady state, gain and absorption coefficients reach their stationary values

$$g = \frac{g_0}{1 + \frac{T_{1g}}{T_R} \Delta N_g}\,, \quad f(\Delta\omega) = \frac{f_0(\Delta\omega)}{1 + \frac{T_{1ab}}{T_R} \Delta \mathcal{N}_a}\,,$$
$$\Delta \mathcal{N}_a(\Delta\omega) = 8 \frac{\tau}{T_{2ab}} \frac{1/3 + (\Delta\omega\tau)^2}{\left[1 + (\Delta\omega\tau)^2\right]^2}\,. \tag{11}$$

It is worth noting that the solution (10) has the same form as for the familiar SIT effect. However, contrary to the classical case, for which the energy/duration of the pulse depends on initial conditions, here the energy/duration has a value uniquely determined

[4] For $K \neq 0$, see [4].

by the balance between net gain and net losses, $g - l_{cav} - l = 0$ (where l is obtained by substitution of (11) into (8)), see Fig. 1. Additionally, now the SIT soliton propagates through the absorber, the properties of which depend on the parameters of the pulse. Thus, frequency distribution of atoms is no longer described by a Gaussian function, but transforms into a more complex shape, compare two curves in Fig. 2. Another, much more important, distinguishing feature between free-propagating and lasing regimes involves the additional stability problems peculiar to all mode-locked lasers. The mere fact of existence of a steady state solution does not assure that this regime can be realized in practice. In order to convince ourselves that the solution is indeed stable and does not break under the action of small perturbations, we have to fulfill additional stability tests. In this Letter we check the stability of the solitary wave of the form of (10) against arising of a weak cw radiation.

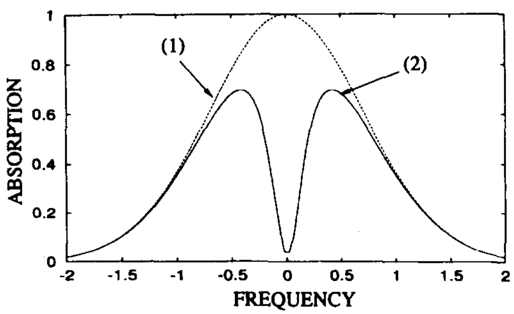

Fig.2. Inhomogeneously broadened contour: (*1*) original Gaussian distribution of absorbing atoms, $f_0(\Delta\omega)$ of width $(T^*)^{-1}$; (*2*) saturated distribution of the atoms in the ground state for the steady-state mode-locked SIT soliton lasing, $f(\Delta\omega)$, obtained from Eq. (11) with $\tau = 6 \cdot 10^{-3} T_{2ab}$. Other parameters are: $T_{1ab} = T_{1g} = 10$ ms, $T_{2ab} = 3 \cdot 10^{-9}$ s, $T^* = 0.44$ ps, $T_R = 660$ ns. Frequency is in units of $(T^*)^{-1}$

In the limit of highly saturated absorber, $f(\Delta\omega = 0) \ll 1$, which is a very good approximation for solid-state and fiber lasers, one can obtain from Eqs. (8) and (11) an expression for the soliton damping rate:

$$l = a_o \left[\frac{T_R}{8T_{1ab}} \frac{\tau}{T_{2ab}} \right]^{1/2}, \quad \tau \gg \max \left[T^* ; 8 \frac{T_{1ab}}{T_R} \frac{T^*}{T_{2ab}} T^* \right]. \tag{12}$$

The center of the saturated line, where the losses for cw radiation reach their minimum, is the most critical region for the pulse stability. The absorption coefficient for a weak cw field at $\Delta\omega = 0$ is defined by Eq. (8) after substituting of $\Delta\mathcal{N}_a^{cw}$ instead of $\Delta\mathcal{N}_a$

$$l_{cw} = \frac{\langle \Delta N_a^{cw}(\Delta\omega) \rangle}{\sqrt{\pi} T^* J} = \frac{3}{8} a_0 \frac{T_R}{T_{1ab}} \frac{\tau}{T_{2ab}}, \tag{13}$$

where we referred to Eq. (4). Stability condition for soliton operation reads as a requirement for soliton losses be smaller than that for cw radiation, $l_s < l_{cw}$. This can be met only with sufficiently short pulses

$$\tau < \left[\frac{9}{8} \frac{T_R}{T_{1ab}} \right]^{1/3} T_{2ab}. \tag{14}$$

Substitution of the experimental parameters (see caption for Fig. 2) in Eq. (14), we get a stable operation for solitons with durations not longer than 140 ps, that corresponds to peak powers more than 1.3 kW. In [3] the pump power was insufficient to produce peak powers as large as this. One can concede this as a reason why the SIT soliton operation was not observed.

In conclusion, the theory of mode-locking of solid-state lasers with inhomogeneously broadened coherent absorber is developed. Stationary solution in the form of the SIT soliton is found and tested for stability against cw perturbations. Only sufficiently short pulses turns out to be stable, see inequality (14). Basing on (14) we conclude that the experimental setup proposed in [3] should be modified to achieve the SIT-soliton lasing.

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