ONE-DIMENSIONAL FERMI LIQUID AND SYMMETRY BREAKING IN THE VORTEX CORE

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Fermions localized within vortex cores can form one-dimensional Fermi liquids. The nonzero density of states in these Fermi-liquids can lead to instability of the symmetric structure of the vortex core. We consider a symmetry breaking which is obtained due to spontaneous admixture of the spin-triplet p-wave component of the order parameter in conventional s-wave vortex in the presence of magnetic field. This occurs at low enough temperature and leads to an asymmetric shape of the vortex core. Similar phenomenon of the symmetry breaking induced by the core fermions occurs in electroweak Z-strings [8].

Introduction. Recently new techniques have been developed, that in principle allow to probe the core structure of the individual Abrikosov vortex in superconductors. This includes a scanning tunneling microscopy [1], electron holography, Lorentz microscopy, etc. (see [2] and Refs. there). That is why the theoretical investigation of the symmetry of the core and of the energy spectrum of the fermions localized in the core is now up-to-date.

Three possible types of the energy spectrum $E(Q, k_z)$ of the fermions localized within the vortex core have been discussed in Ref.[3]. (i) The spectrum $E(Q, k_z)$, where Q is the generalized angular momentum, has a gap on order Δ^2/E_F [4] (here Δ is the superconducting gap and E_F is the Fermi energy in the normal Fermi liquid). (ii) One or more branches of the spectrum with particular Q's cross the zero level as functions of the linear momentum k_z at some points $k_z = k_{F,Q}$. In this case fermions occupying negative energy levels form one-dimensional (1D) Fermi liquids with the Fermi points at $k_{F,Q}$ (see also an example of the continuous vortex in ^{3}He-A [5]). (iii) The flat band where all fermions have exactly zero energy in the finite region of the momenta k_z, which is called the Fermi condensate [6].

It was emphasized that both the 1D Fermi liquid and 1D Fermi condensate can be unstable towards a further breaking of the vortex symmetry at low temperatures, in particular the vortex can become nonaxisymmetric or the vortex line can transform into a spiral [3] (see also [7]).

Recently such kind of instability of the vortex caused by the fermion zero modes in the core has been discussed for the Z-string solution in the Weinberg-Salam model of electroweak interactions [8]. In this case the situation corresponds to the case of 1D Fermi liquid. While in the Ref.[8] it was stated that this can lead to the absolute instability of the Z-string, the situation is actually not dramatic for the existence of the locally stable Z-string and is similar to that which has first been discussed by Peierls [9]. In the Peierls model the 1D electron liquid coupled to the lattice is unstable at low temperature towards the dimerization of the lattice. In the Z-string the 1D massless relativistic fermions coupled to the bosonic field of the order parameter (Higgs field) lead below some critical temperature to the spontaneous symmetry breaking, which is realized, e.g., in the

development of the small amount of the upper component of the Higgs field or of another mode of instability. The transition temperature $T_{c\,core}$ of such instability is usually exponentially small.

We discuss here the similar instability for the condensed matter vortices. We have found that in most cases the transition temperature $T_{c\,core}$ is extremely small of the order $\Delta \exp(-E_F/\Delta)$, but in some cases $T_{c\,core}$ can be reasonable, of the order Δ^2/E_F.

s-wave superfluids. Let us consider as an example the situation in super-conductors with s-pairing. The dependence of the order parameter on coordinates around the axis of one-quantum vortex is given by $\Delta(r) \cdot \exp(i\phi)$ in cylindrical coordinates. The magnitude of the gap $\Delta(r)$ equals zero at the axis $r = 0$ and tends to the bulk value Δ for large distances $r \gg \xi$, where $\xi = v_F/\Delta$ is the coherence length.

The system is invariant under translations along the axis z. If the spin-orbit interaction is neglected there is also the $SU(2)_S$ group of spin rotation, which is reduced to $SO(2)_S$ subgroup if the magnetic field is applied. The "axial" symmetry of the order parameter in the vortex is characterized by another $SO(2)_Q$ with a generator which is the combination of the orbital momentum $\mathbf{L}_z$ and the particle number operator $\mathbf{I}$ [10]:

$$Q = \mathbf{L}_z - \frac{m}{2}\mathbf{I} \tag{1}$$

where $m = 1$ is the winding number of the vortex. The fermion eigenstates are characterized by the quantum numbers k_z, Q and the direction of the spin ($\uparrow$ or $\downarrow$). The energy spectrum of low-lying excitations is given by [4]:

$$E(Q, k_z) = Q \cdot \omega_0(k_z) \tag{2}$$

with $\omega_0 \sim \Delta^2/E_F$ for $k_z \ll k_F$ and $\omega_0 \to \infty$ for $k_z \to k_F$ (this approximation is true for $\omega_0 \ll \Delta$) [4]. For $m = 1$-quantum vortex Q can assume only half-integer values (L_z is integer, and I is $+1$ and -1 for particles and holes correspondingly).

Let us apply a magnetic field H parallel to the axis of the vortex. Then the energy spectrum is shifted down (up) for fermions with spin up (down):

$$E_{\uparrow,\downarrow}(Q, k_z) = Q \cdot \omega_0(k_z) \mp \mu H, \tag{3}$$

and we get the picture (fig.1) where the branches $E_{Q,\uparrow}(k_z)$ and $E_{Q,\downarrow}(k_z)$ intersect for $\mu H > Q\omega_0(0)/2$.

The fermions near the points $k_{F,Q}$ can be considered as a one-dimensional Fermi-liquid, and $k_{F,Q}$ plays the part of the Fermi-momentum. The nonzero density of states can lead to a new pairing of the one-dimensional fermions, the character of which depends on the sign and the magnitude of the interaction of 1D fermions in different channels. Let us show that both symmetries $SO(2)_S$ and $SO(2)_Q$ will be broken at low enough temperatures, if the interaction in the spin-triplet p-wave channel is attractive, and the core of the vortex can become anisotropic.

p-wave component in the core. In our model an extra interaction of the fermions in 1D Fermi liquid results from the interaction of the initial bare fermions in the p-channel and we are looking for the instability of the vortex core towards nucleation of the admixture of the spin triplet p-wave component in the core of the type:

$$\delta\hat{\Delta} = g \; \Delta(\mathbf{d} \cdot \hat{\vec{\sigma}})i\hat{\sigma}_2(\mathbf{a} \cdot \mathbf{k}) \; f(r)\exp(iN\phi) \; . \tag{4}$$
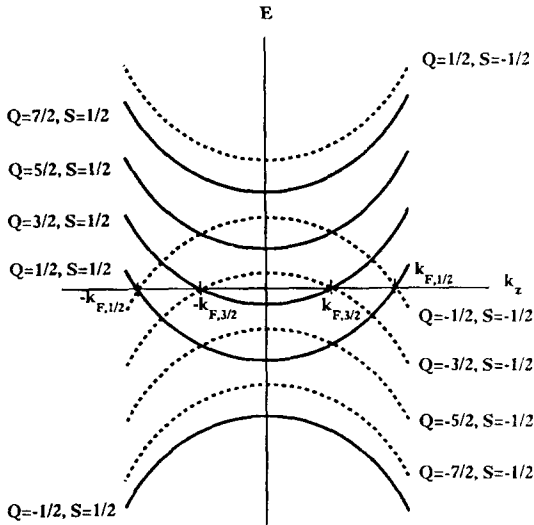
Fig.1. Spectrum of fermions in the vortex in *s*-wave superconductor. The levels are shifted by magnetic field. Some branches cross the zero energy level forming 1D Fermi liquid. The fermions with opposite spins on the Fermi level are paired at low T which leads to broken symmetry of the core

As in Ref.[8] we assume that $f(r)$ is concentrated in the core since it should influence only the core fermions. $f(r)$ is of the order of unity in the core and decreases to zero outside the core of the vortex (i.e., at $r \sim \xi$); in addition $f(0) = 0$, if $N \neq 0$. The parameter Δ is the bulk value of the *s*-wave gap and the dimensionless parameter g is thus the magnitude of the admixture of the *p*-wave order parameter in the core region as compared to the *s*-wave component. For simplicity we can assume a trial function of the form $f(r) = \exp(-r/\xi)$ for $N = 0$. The spin vector d should be orthogonal to the magnetic field to break the spin rotation symmetry.

The suggested change of the order parameter breaks Q-symmetry as well. To show this note that $\delta\hat{\Delta}$ is the sum of three orbital harmonics: $\mathbf{a} \cdot \mathbf{k} = a_+k_- + a_-k_+ + a_0k_z$ where $k_\pm = k_x \pm ik_y$. We suppose that only one a is nonzero, namely a_l with some value $l = 0, \pm 1$ of the z-projection of the Cooper pair orbital momentum. It is easy to check that new order parameter is not Q-invariant for $N + l \neq 1$ and this means the violation of the axial symmetry of the whole core structure. g is the dimensionless parameter which shows the strength of the deformation. This symmetry breaking leads to mixing of $Q, \uparrow$ and $-Q, \downarrow$ fermionic states in the same way as u and d quarks are mixed in the Z-string in the Naculich scenario [8] of the instability of Z-string. Due to the noncrossing theorem for the levels of the same symmetry, the two branches which crossed each other at $k_{F,Q}$ repel each other and the gap between the states appears which is proportional to g.

Deformation of the fermionic spectrum. Without the admixture the Bogolyubov-Nambu hamiltonian is given by 4×4 matrix

$$\mathcal{H}_0 = \begin{pmatrix} \xi & 0 & 0 & \Delta \\ 0 & \xi & -\Delta & 0 \\ 0 & -\Delta^* & -\xi & 0 \\ \Delta^* & 0 & 0 & -\xi \end{pmatrix}. \tag{5}$$

Here $\xi = (1/2m^*)(-d^2/dr^2 - k_F^2)$. The eigenstates of $\mathcal{H}_0$ with energies (2) for spin up and distances $r \gg k_F^{-1}$ are given by [4]

$$
\chi_{k_z,Q,\uparrow} = \begin{pmatrix} \chi_1 \\ 0 \\ 0 \\ \chi_2 \end{pmatrix} = const \cdot e^{ik_z z} e^{iQ\phi} H_\nu(qr) e^{-K(r)} \cdot \begin{pmatrix} \exp\left(\frac{i}{2}(\phi + \psi(r))\right) \\ 0 \\ 0 \\ -i \exp\left(-\frac{i}{2}(\phi + \psi(r))\right) \end{pmatrix} \quad (6)
$$

where H_ν is the Hankel function with index $\nu = \sqrt{Q^2 + 1/4}$; $q^2 = k_F^2 - k_z^2$,

$$
K(r) = \frac{m^*}{q} \int_0^r \Delta(r')dr',
$$

$$
\psi(r) = -\int_r^\infty \exp(2(K(r) - K(r'))) \left(\frac{2Em^*}{q} + \frac{Q}{qr'^2}\right) dr'.
$$

For spin down the eigenstates are

$$
\chi_{k_z,Q,\downarrow} = \begin{pmatrix} 0 \\ \chi_1 \\ -\chi_2 \\ 0 \end{pmatrix}. \quad (7)
$$

The change $\delta\hat{\Delta}$ leads to the perturbation of the mean field Bogolyubov-Nambu hamiltonian. The angular integral in the matrix element of this perturbation between the states $\chi_{k_z,Q,\uparrow}$ and $\chi_{k_z,-Q,\downarrow}$ does not vanish only for

$$
N + l + 1 \pm 2Q = 0 , \quad (8)
$$

(i.e. $N + l$ should be an even integer). In this case the matrix element between the spin-up and spin-down states is of the order of $g\Delta$. For other Q the integral over $d\phi$ in the matrix element vanishes. The 2×2 hamiltonian for two mixing states $(k_z, Q, \uparrow)$ and $(k_z, -Q, \downarrow)$ for k_z near $k_{F,Q}$ is then given by:

$$
\begin{pmatrix} E_{old} & g\Delta \\ g\Delta & -E_{old} \end{pmatrix}. \quad (9)
$$

Here $E_{old} = v_{F,Q}(k_z - k_{F,Q})$, the derivative $v_{F,Q} = Q\omega_0'$ of the energy over momentum k_z represents the "Fermi" velocity of the 1D Fermi-liquid. The modified energy spectrum takes the form

$$
E_{new} = \pm\sqrt{v_{F,Q}^2(k_z - k_{F,Q})^2 + g^2\Delta^2}. \quad (10)
$$

At zero temperature fermions occupying the states with negative energy near $k_{F,Q}$ gain in energy after the change in the order parameter. We are going now to compare energy gain and energy losses due to the perturbation g.

Energy balance and zero-temperature deformation of the vortex core. The main contribution to the difference between the energy of the fermionic vacuum in

symmetric and nonsymmetric states comes from the logarithmic term which arises due to appearance of the energy gap in 1D Fermi-liquid:

$$E_{gain} = \int \frac{dk_z}{2\pi}(E_{new} - E_{old}) \cong -\frac{g^2\Delta^2}{v_{F,Q}} \int_{k_1}^{k_2} \frac{d(k - k_{F,Q})}{|k - k_{F,Q}|} \ . \tag{11}$$

The lower cut-off is determined by the gap itself

$$k_1 \cong \frac{g\Delta}{v_{F,Q}} \ . \tag{12}$$

The most interesting situation takes place for $k_{F,Q}$ not too close to k_F, in this case the Fermi-velocity $v_{F,Q}$ is small and the energy gain increases. This occurs in field H near its threshold values $H_c(Q)$ at which the branches $Q,\uparrow$ and $-Q,\downarrow$ start to cross each other and the "Fermi-momentum" $k_{F,Q}$ appears for the first time. The "Fermi-momentum" $k_{F,Q}$ is given by the condition $Q\omega_0(k_{F,Q}) = \mu H$, that is we are interested in the fields $\mu H \sim Q\Delta^2/E_F$. In this region the upper cut-off in the logarithmically divergent integral in (11) is $k_2 \sim k_{F,Q} \sim k_F\sqrt{\frac{\mu H}{Q\omega_0(0)} - 1}$. This leads to the result

$$E_{gain} \cong E_F k_F g^2 \frac{k_F}{|Q|k_{F,Q}} \ln\left(\frac{|Q|\Delta k_{F,Q}^2}{g E_F k_F^2}\right) . \tag{13}$$

On the other hand the energy loss due to the perturbation of the superconducting order parameter in the core of the vortex is of the order of $E_{loss} \cong E_F k_F g^2/\gamma_1$. Here γ_1 is the relative magnitude of the attractive interaction of bare fermions in the p-channel compared to that in s-channel. For $N \neq 0$ the contribution to the gradient energy of space inhomogeneity of $\delta\Delta$ increases by the amount $\sim E_F k_F N^2 g^2$. This consideration is valid only if the latter term is smaller than the former one.

Comparison of the energies shows the instability, the relative magnitude g of the new p-wave order parameter at zero temperature being of the order of

$$g \cong \frac{\Delta}{E_F}|Q|\left(\frac{k_{F,Q}}{k_F}\right)^2 \exp\left(-|Q|\frac{k_{F,Q}}{k_F}(\frac{1}{\gamma_1} + N^2)\right) \ . \tag{14}$$

Here we omitted the coefficients of the order of unity. $N(Q)$ is given by (8). At small Q the exponent $\exp(-O(1))$ is not too small for $k_{F,Q} \sim k_F$, and the transition temperature is reasonable. On the other hand the numerical factor in the exponent $O(1)$ can be large. In this case (even for large Q) we can take $k_{F,Q}/k_F$ to be small which leads to the exponential increase in the magnitude of g followed by just power-law decrease in the preexponential factor.

Symmetry breaking scheme. The above result means that there should be a second-order phase transition at $T_{c\,core}$ into the broken symmetry state. This is the state with the additional superfluid order parameter (4) with p-pairing. The k-dependence of this order parameter is described by the quantum number $l = 0, \pm 1$, and the spatial ϕ-dependence by another integer $N = \pm 2Q - l - 1$. So, there are six competing structures (for three values of l) with comparable transition temperatures. They correspond to different symmetry groups of the new vortex core structure below $T_{c\,core}$.

The spin structure of the new order parameter is described by the vector d in the xy-plane. For $d = \hat{x} \pm i\hat{y}$ the symmetry $SO(2)_Q \times SO(2)_S$ is reduced to the combined symmetry group $SO(2)_{Q \pm 2|Q \pm 1|S}$. For other d in the plane the symmetry is reduced to the discrete group $Z_{4|Q \pm 1|}$ the elements of which are Q-rotations accompanied by the possible inversion of d. In the former case, say, density is still axially symmetric, and in the latter case it is not.

Transition temperatures. The temperature $T_{c\,core}$ at which the transition to the asymmetric core state occurs is

$$T_{c\,core} \sim g\Delta \ . \tag{15}$$

(see the phase diagram, fig.2). The maximal transition temperature in the Q-th region and the range of the fields where the bare structure is unstable (the height and the width of the region in $H - T$-plane) decrease with Q:

$$T_{c,max}(Q) \sim \Delta \mu H(Q) \sim \frac{\Delta^2}{E_F} \frac{1}{Q^5}. \tag{16}$$

The minimal field is of order $(\Delta^2/E_F)/\mu$. This field can be less than the upper critical field H_{c2} if the effective fermion mass $m^\star$ is larger than the bare electron mass m.
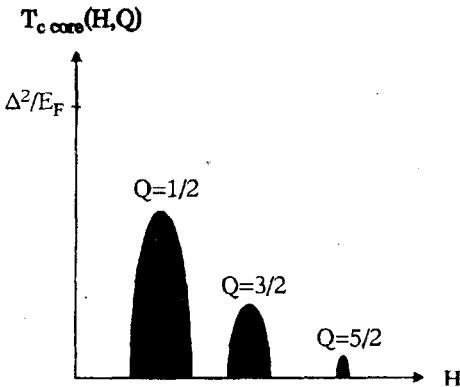

Fig.2. Schematic dependence of the temperature of the core transition on the magnetic field. Within the shaded area the symmetry of the vortex core is broken
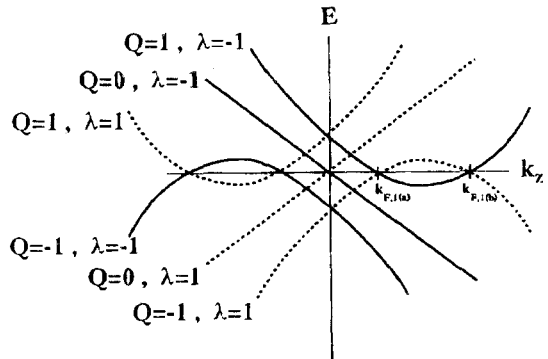
Fig.3. Spectrum of fermions in the most symmetric vortex in ³He-B [3]. $\lambda = \pm 1$ is helicity

Vortex in ³He-B. In the spin-triplet superfluid ³He-B the situation is almost equivalent to that in s-superfluids. In the most symmetric B-phase vortex the branches of the energy spectrum are described by quantum numbers k_z, Q and helicity $\lambda = \pm 1$ [3]. The branches $E_{Q=0,\lambda=1}(k_z)$ and $E_{Q=0,\lambda=-1}(k_z)$ intersect at the the point $k_z = 0$, $E = 0$ even in zero external magnetic field. The branches $E_{Q,\lambda=1}$ and $E_{-Q,\lambda=-1}$ begin to intersect at $k_z \sim k_F$ in the fields $\mu H_c(Q) \sim Q\omega_0 \sim Q\Delta^2/E_F$ (fig.3).

In the case there is no conservation of longitudinal spin component S_z, and the perturbing order parameter may have vector d of spin anisotropy parallel to z-axis. The deformation of the vortex can occur within the triplet pairing state; the singlet pairing (e.g., s-pairing) is accepted as well.

Transition temperatures, critical fields and the whole phase diagram are more or less the same as for s-superfluids. For the two branches mentioned above which intersect in zero field the transition temperature is of the order of $T_c \sim \Delta^2/E_F$. It was found experimentally and theoretically that the most symmetric vortex in ^{3}He-B is unstable towards the symmetry breaking at least at high temperature of order T_c [10, 11, 12]. Thus if the symmetry is restored at lower temperature it is broken again at very low T.

The states with definite λ are the mixtures of states with different particle number and different $S_z = \pm 1/2$. The analysis shows that in the Q-th region the angle dependence of the perturbation of the order parameter can be given by an integer

$$N = \pm 2Q - l - m + \Sigma$$

where $\Sigma = 0, \pm 2$. Therefore for each l we get three different values of N and thus several competing types of the symmetry breaking.

Conclusion. The vortex structure, which contains the one-dimensional Fermi-liquid of the core fermions, is unstable towards the symmetry breaking. In the case of Abrikosov vortex in the s-wave superconductors this leads to the breaking of the axial symmetry of the vortex core in some regions in $H - T$-plane.

Similar symmetry breaking occurs in the electroweak Z-string, where the mixing of the d and u quarks opens the gap in the fermionic spectrum [8]. In the Abrikosov and the ^{3}He-B vortices this corresponds to the hybridization of $Q, \uparrow$ and $-Q, \downarrow$ states of fermions. For the Abrikosov vortex this leads to appearance of the p-wave pairing component in the core.

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