Pis'ma v ZhETF, vol.63, iss.3, pp.150 - 154 © 1996 February 10

SYMMETRY BREAK-UP IN DISTRIBUTED COUPLING OF COUNTERPROPAGATING WAVES IN A SUPERRADIANT LASER

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Submitted 13 December 1995

The instability of superfluorescent decay is investigated for the case in which sample length is greater than or comparable with the resonant wavelength. It can inhibit emission in the direction of the pump pulse and even cause the forward emission to be switched off.

PACS 42.50.Fx, 42.50.Ne, 42.65.Pc

The recent experiments on the Dicke superfluorescence (DSF) in a solid-state sample [1] have conjectured that the distributed coupling between counter-propagating waves may trigger superfluorescent radiation into a spatial mode operating as a distributed-feedback superradiant laser. These results have followed by the theoretical study where the retarded coherent response mixes the left- and right-running waves of radiation thus creating a local interference pattern [2]. However this approach closely related to the earlier concept of Ref. [3], has the disadvantage that it can be hard to extract general information from specific numerical results. An alternative approach exploits the simplest model of counterpropagating pulses [4, 5], but the physical mechanism behind the linear coupling between the waves has not been identified.

The objective of this Letter is to develop the semi-classical model of DSF, including both spatial-temporal dynamics of the radiation field and dynamic properties of two-level quantum emitters and is based on the following argument. When, owing to the quantum fluctuations of dipole moment and field, an inverted emitter decays down, it emits independently on the rest of an ensemble, the intensity of the radiation from the sample is a sum over all emitters, and no correlation occurs [6]. Later on, the non-zero polarization of a single emitter seeds a regular, classical field which is fed by the regular fields of surrounding emitters. The reaction of a two-level emitter on the on-resonance external field $E_{ext} = E_0 \exp(\imath\phi_{ext})$ can be described in terms of the Bloth angles θ and φ,

$$\frac{d\theta}{dt} = \frac{2\mu E_0}{\hbar}\cos\left(\phi_{ext} + \frac{\pi}{2} - \varphi\right), \tag{1}$$

$$\frac{d\varphi}{dt} = \frac{2\mu E_0}{\hbar}\cot\theta \cdot \sin\left(\phi_{ext} + \frac{\pi}{2} - \varphi\right). \tag{1b}$$

where μ is the matrix element. As can be readily seen from Eqs. (1a) and (1b) the characteristic time of the relaxation of the emitter phase φ to its equilibrium value $\phi_{ext} - \frac{1}{2}\pi$ is equal to $\tau_\varphi \simeq \frac{\hbar}{2\mu E_0}\tan\theta$ whereas that of the polarization θ is

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$\tau_\theta \simeq \hbar/2\mu E_0$. The ratio $\tau_\varphi/\tau_\theta \simeq \tan\theta$ allows one to write $\tau_\varphi \ll \tau_\theta$ whilst the emitter remains near the state of inversion where $\theta \approx \pi$. This can be interpreted such as the collective field locks phases of all emitters creating this field, and only afterwards the energy of excitation is taken away by the wave of the polarization θ. The mechanism of appearance of the feedback structure can then be thought as the result of the interference between the source and its own regular field backscattered by an inverted environment. Consequently, an over-all picture of the process must be described without resorting to the slow-envelope approximation in which the action of the backscattered wave is neglected.

First of all, relevant expressions from the theory are abstracted and adapted. Near the state of inversion, the polarization of the single emitter is governed by the equation as follows [3, 7],

$$\frac{\partial P(x,\tau)}{\partial \tau} = \frac{1}{L}\int_{-L/2}^{L/2} P(\xi,\tau)\exp(\imath k_0|x-\xi|)d\xi, \tag{2}$$

where $\tau = (2\pi\rho L k_0 \mu^2/\hbar)t$ and L is the sample length. Eq. (2) implies both the effect of the self-reaction which is due to the own field of the emitter and is responsible for its spontaneous decay, and the retarded response as a fast argument $\sim \exp(\imath\omega|x_l - x_j|/c)$.

The right-hand side of Eq. (2) has a non-Hermitean kernel that generates an infinite discrete set of complex eigenvalues for the system under consideration. Such a system, being initially entagled as a superposition of, say, two eigenstates with close eigenfrequencies, can display a beating behavior as a pair of coupled mechanical oscillators. By means of the truncation procedure described elsewhere [8], one can simplify the analysis and express the output intensity in terms of the sample length and resonant wavelength,

$$I_{r,l} = \frac{|A_r(0)|^2}{2}e^{\tau}\left[\cosh\left(\tau\frac{\sin k_0 L}{k_0 L}\right) \pm \cos\left(\tau\frac{\cos k_0 L}{k_0 L}\right)\right]. \tag{3}$$

This equation displays that the coupling of the right- and left-running waves is extremely sensitive to the exact length of the sample, small changes can have dramatic consequences.

Indeed, if $k_0 L = \pi m$ i.e. the integer number of the half-waves lies over the sample length, Eq. (3) shows that on approaching its local maximum in one direction the radiation vanishes on the opposite side of the sample, but the total output $I_r + I_l$ grows. Moreover the radiation in forward direction can, in principle, be switched off. Such a photonic antenna-like action [9] stems from the coherent excitation of two coherent eigenstates. In dimensional units, the frequency of these oscillations is $\omega_{osc} = 2\pi\rho\mu^2/\hbar$, and owing to the equal increment for both states, the amplitude of oscillations equals unity.

Otherwise, at $k_0 L = \pi/2 + \pi m$, Eq. (3) shows that the intensity experiences monotonous growth for both directions. Here the coherent pump prepares the sample in the state described by a single eigenvalue which, in turn, defines the increment of the decay. The intensities on the opposite sides remain equal at $t \geq \hbar/2\pi\rho\mu^2$ until nonlinear effects raise to dominate the process.

This analysis is now used to explore the nonlinear stage of the decay. In what follows, the set of the Maxwell-Bloch equations written for the field E, angles θ and φ is integrated numerically. By preceding these results, it is worth to explain the choice of the initial conditions for θ and φ. Setting the amplitude of the regular field, both of the surrounding emitters and of its own, equal to the emitter's fluctuating field, yields [10]

$$\theta_0(x, t=0) \simeq \frac{1}{\sqrt{\rho\lambda^2 L}},$$

and to provide the non-correlated initial state the phase $\varphi(x, t=0)$ is chosen to be a random function of x; at $t=0$ electric field is obviously $E(x, t=0) = 0$.

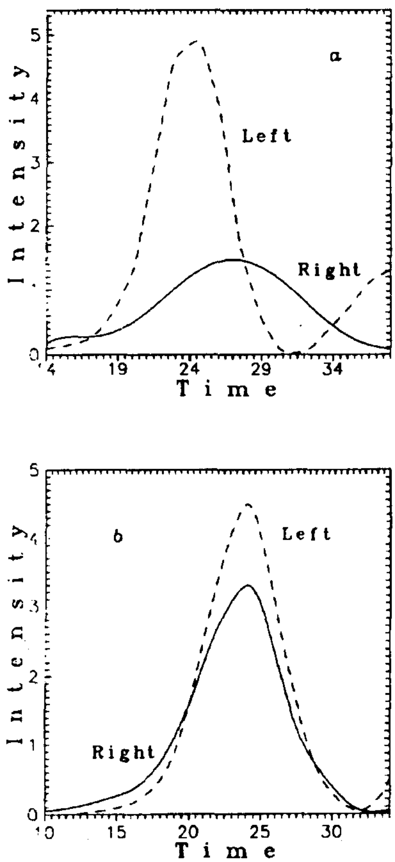

Fig.1. The DSF main pulse from the sample of $L = 10\lambda$ and $L = 10.25\lambda$, plots (a) and (b) correspondingly, vs normalized time, $\rho\lambda^3 = 10^5$

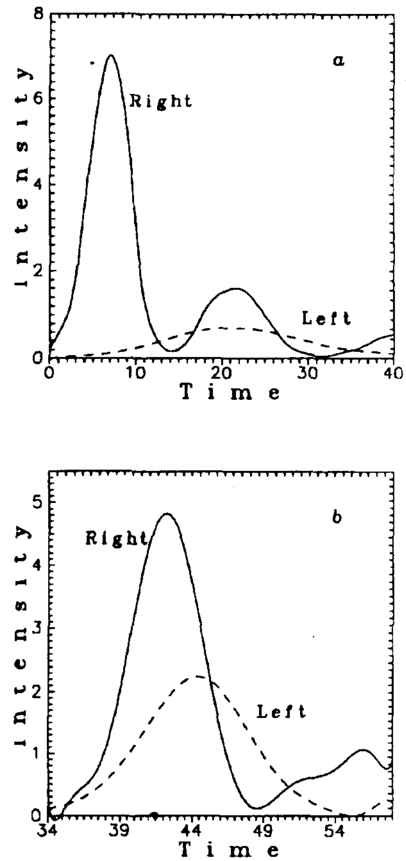

Fig.2. The same as in Fig.1a, but for $\rho\lambda^3 = 10$ (a), and $\rho\lambda^3 = 10^7$ (b)

In the experiments of the Düsseldorf's group [1] $\rho\lambda^3 \simeq 10^5$, and this choice gives $\theta_0 = 10^{-3}$. An ultrashort pump pulse penetrates the sample from the left and inverts the two-level medium which length is taken to be $L = 10\lambda$; thus ensuring the fulfillment of the above conditions of the distributed coupling. The simulations suggest that this is the case for the set of chosen parameters as shown in Fig.1a. The main share of the DSF radiation escapes the sample in the backward direction, while the forward running pulse is considerably weaker. This does not occur for the sample of $L = 10.25\lambda$ in where the coupling is suppressed, hence keeping the sample as a normal bidirectional laser (Fig.1b). It is also seen that the analytic solution (Eq.3) is in a fairly good agreement with numerical one.

Fig.2 confirms the statement of the linear theory that the level of the asymmetry is sensitive to the density. Thus, for $\rho\lambda^3 = 10$ the oscillation can not occur by the time $t = \tau_d$, $\omega_{osc}\tau_d \ll 1$, and the main pulse is a right-running (Fig.2a), in the preferred direction defined by the excitation. On increasing $\rho\lambda^3$ up to 10^7, one can pass through the entire cycle of the alternation of the DSF direction, $\omega_{osc}\tau_d \gg 1$, and yield the main pulse running from the left to the right boundary once again (Fig.2b).

Fig.3a shows the spatial distribution of the inversion inside the sample of $L = 10\lambda$ for two subsequent moments of time. Here, in accordance with the linear approximation, the DSF can be suppressed in the forward direction. It is seen the switching is accomplished by patterning a grating on the inversion profile, and the period of the grating equals to $\lambda/2$ in agreement with previous calculations [2, 8]. On the nonlinear stage of the decay, the inversion is a decaying sequence of spikes reflected from the edge back into the sample. Every spike of this sequence propagates with its own velocity, and their collisions demolish the inversion completely. Again, tiny changing the length by $\lambda/4$ leads to a distinguished evolution of the inversion which is, in principle, the energy of the excitation (Fig.3b). On the linear stage, the calculations show the grating grows near the left edge of the sample and is localized within the length $kl_{loc} \simeq \pi$; it then moves without spreading, and the inversion has, on average, a symmetric shape. A significant feature in Fig.3b is an inversion trapping in the middle of the sample, which occurs whenever the main pulse comes over its maximum. The inversion takes a hump-like shape and loses its energy by emitting pulses walking away to the sample boundaries. When the width of the hump approaches $\lambda/4$, this emission finishes, and the inversion remains localized until the fluctuations eventually drain it out within the time $\approx T_1$.

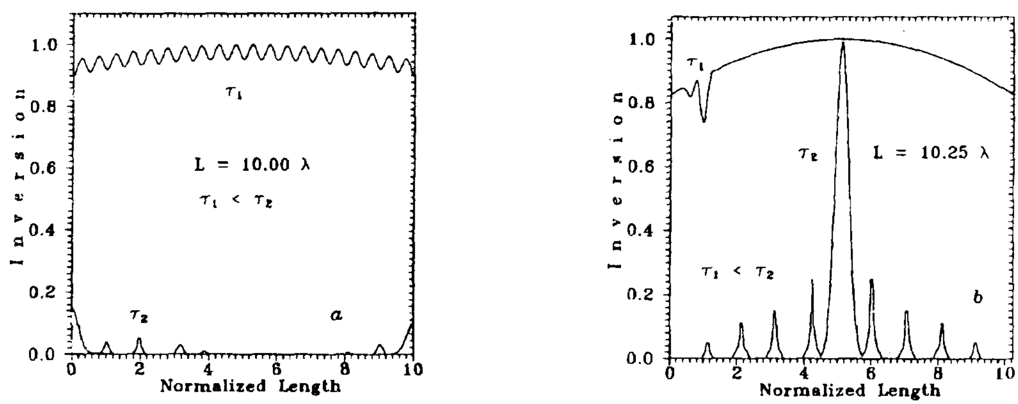

Fig.3. Grating formation, inversion demolishing and trapping; the parameters are $L = 10\lambda$ (a) and 10.25λ (b), $\rho\lambda^3 = 10^5$

In a conclusion, it should be noted that a density- and length-dependent switching is observed for DSF in a solid-state sample whose length exceeds the resonant wavelength. In particular, the radiation may be reflected into the backward direction, relative to the pump pulse. This result can be exploited in the design of a new type directing antenna [9] or a superradiant laser [11]. In addition, it is observed that the energy of excitation may be trapped within a small region after the termination of a superradiant laser.

This work was supported by The Royal Society, London. The author would like to thank A.V.Karnyukhin for his kind and insistent suggestions that he considers this problem; he also acknowledges F.Haake and J.W.Haus for many interesting discussions and critical reading the manuscript.

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