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SPIN GAP FOR MULTICHAIN LATTICE AND LINEARIZED NEAR FERMI POINTS QUANTUM SPIN MODEL

A.A.Zvyagin[1)]

B.I. Verkin Institute for Low Temperature Physics and Engineering of the National Ukrainian Academy of Sciences
310164 Kharkov, Ukraine

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We have shown using exact Bethe ansatz solution that for a multichain quantum antiferromagnetic spin $S = 1/2$ system an excitation energy has a gap for the linearized near Fermi points case of the model and it is gapless for its lattice analog. The result is generalized for multichain spin S quantum antiferromagnet and for strongly correlated electron models with "permutation" construction of supersymmetric Hamiltonians.

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During last years the interest for the low-dimensional quantum spin systems has been renewed. This interest is connected with, e.g., the question whether low-lying spin excitations have gaps or they are gapless. The well-known Holstein–Primakoff transformation from spin operators to Bose ones (creation and annihilation of spin waves) within the linear response in $1/S$ (S is the site spin value) predicts gapless spin excitations for $D = 1, 2$ isotropic Heisenberg antiferromagnets [1, 2]. But, in fact, this linear response scheme works well for the ordered phases, for which the number of spin deviations at each site is much smaller than S; it does not work, e.g., for the $S = 1/2$ case in low dimensional spin models, where the ground state fluctuations are enhanced. Various mean field theories (including the functional integration stationary phase approach for the nonlinear σ-model, see, e.g., [3]) give a gapless (ordered) phase for the 2D quantum isotropic spin antiferromagnet: the critical spin value below which the disordered "quantum" phase emerges is less than $1/2$ [4]. For the 1D quantum spin antiferromagnets the field theory predicts qualitatively different results for integer and half-integer site spin values: the integer-spin Heisenberg chains have spin gaps for low-lying excitation spectra, but half-integer ones have no gap, due to the so-called θ-vacuum topological term in the Lagrangian [2, 5]. It is obvious that a mean field theory cannot in principle give correct answer to this essential question, thus one has to use exact methods. One of the most powerful methods of solving the 1D quantum systems exactly is the algebraic Bethe ansatz (or the quantum inverse scattering method), see, e.g., [6]. Using the Bethe ansatz method one obtains the gapless behavior for the $S = 1/2$ Heisenberg antiferromagnet [7] in agreement with the field theory result [2, 5] (the same answer holds for the $SU(2)$ spin S 1D antiferromagnets with permutation operators for the Hamiltonians [8]). We have to note here that there exists a different (but, also, exact) approach with projection operators in Hamiltonians, which gives gapful excitations for integer spin systems [9]. Unfortunately, the exact answer for a quantum 2D antiferromagnet has not been known yet. One of the ways to obtain some special features with

1) e-mail: zvyagin@ilt.kharkov.ua

the help of exact 1D methods for the 2D space is to use the mapping of the 1D exact solutions, e.g., in a field theory, onto the 2D case. More than one decade ago, Polyakov and Wiegmann [10] solved the $(1+1)$ classical nonlinear σ-model problem by mapping it onto the chiral 1D fermionic field theory. They have shown that for an isotropic classical magnet low-lying excitations have gaps, but for the nonlinear σ-model with the topological Wess–Zumino term the gap vanishes. Recently it has been shown exactly [11] that for the multichain spin 1/2 model with T and P symmetries violation (with the topological term in the Hamiltonian, so-called antichiral spin model) low-lying excitations are gapless. It was conjectured that the gapless behavior is the consequence of that topological θ-vacuum term presence. The multichain quantum problem is interesting because it has some characteristic features of both 1D and 2D spaces, and the interest for this problem has been growing over the last few years, since it is connected with the so-called quasi-1D ladder systems, [12]. Furthermore a lot of theories have studied the multichain quantum problem independentely [13].

In this note we would like to mention a new interesting fact, which could throw some light on the problem of low-dimensional quantum spin model excitations. Namely, we show here that depending on the order of taking long-wave linearisation limits near Fermi points and solving the Schrödinger equation (exactly) for the multichain quantum antiferromagnet one obtains drastically different results for the spin excitation gap: the lattice model excitation energy is gapless, but the linearized near Fermi points long-wave Hamiltonian has a gap.

Let us start with the spin 1/2 multichain model transfer matrix $\mathcal{T}$. It was shown in [11], that it has the form

$$\mathcal{T}(\lambda) = \prod_{r=1}^{2L} T(\lambda - \theta_r), \tag{1}$$

where λ is the spectral parameter, $2L$ (even) is the number of spin chains, $T(\lambda)$ is the usual transfer matrix for one spin 1/2 chain, and θ_r are the interaction constants. Using the quantum inverse scattering method we have shown that the Bethe ansatz equations for the rapidities λ_j, which parametrize the eigenfunctions and eigenvalues of the transfer matrix have the form:

$$(\prod_{r=1}^{2L} f(2(\lambda_j - \theta_{r,1})))^N = \prod_{k=1}^{M} f(\lambda_j - \lambda_k), \tag{2}$$

where N is the number of spins in each chain, M is the number of down spins, $f(x) = (x+i)/(x-i)$, and $\theta_{r,1} = \theta_r - \theta_1$. It could be easily seen that the Bethe ansatz equations have the same structure as ones for the classical nonlinear σ-model [10]. It is worth noting, that the number of spin chains ($2L$) works for chiral field theory like the number of chiralities (L is the number of different opposite Fermi points), thus the $2L \to \infty$ limit corresponds, both for field theory and for multichain problem to the 2D case. In [11] we constructed the Hamiltonian of the system, as usual for lattice quantum inverse scattering method [6], as a logarithmic derivative of the transfer matrix Eq. (1),

$$\mathcal{H} = -i(\mathcal{T}(\lambda))'(\mathcal{T}(\lambda))^{-1}|_{\lambda=0},$$

therefore the eigenvalues of such a lattice quantum spin 1/2 Hamiltonian were equal to

$$E_{lat} = -\frac{1}{2}\sum_{r=1}^{2L}\sum_{k=1}^{M}\left((\lambda_k - \theta_{r,1})^2 + \frac{1}{4}\right)^{-1} . \tag{3}$$

One can find the lattice Hamiltonian written in spin operators in [11]. Naturally, according to Eq. (3), and like other exactly solved lattice isotropic $SU(2)$ models [6, 8], the lowest spin excitation (the so–called spinon, which carries spin 1/2 and nonzero spin chirality) has no gap

$$E_{lat}^{exc} = \pi\sum_{r=1}^{2L}\mathrm{sech}(\pi(\lambda_0 - \theta_{r,1})),$$

where λ_0 is the spinon's rapidity. Note, that due to the periodic boundary conditions the half-integer spin excitations confine into integer spin ones, like for one spin chain [7], but with zero spin chirality. Really, since taking the long-wave limit of the energy E_{lat}^{exc} cannot produce a gap, it is still gapless.

But for field theory models [10] (see, also, Ref. [14], where the multiindex classical and quantum magnet problem was solved exactly with the explanation of the connection between the multiindex spin problem with the field theory [10]) the Hamiltonian is constructed in a just different scheme, namely, from the very beginning the Hamiltonian is the linear form in quasimomenta near the Fermi points (plus, naturally, interaction),

$$\mathcal{H} = -i[\log \mathcal{T}(0) - \log \mathcal{T}(\theta)]$$

(θ was the coupling constant). We have to remind that the logarithm of transfer matrix is usually defined as the total quasimomentum of a system for Bethe ansatz problems [6]. On the other hand, it is well–known that the ground state of any spin 1/2 model is constructed like a Fermi sea filling [6] (there may exist either 1 or 0 spin flips for one site, or for one λ_j, like fermions): the ground state corresponds to the occupation numbers of such fermion excitations with negative energy (renormalized or "dressed" [6] due to interaction) equal to unity, and with positive energy equal to zero, like free Fermi gas. Fermi points (we are in 1D, Fermi sphere for 2D) are as usually defined as the points in λ distribution for which dressed energies become zero, see [11]. We construct now the linearized near Fermi points Hamiltonian (it is similar to usual linearization method used in the Abelian bosonization, see, e.g., [15], for Luttinger liquids [16], see, also, [13] for multichain case) of our lattice multichain system in the same way as in [10], (let us assume, that $\theta_{r,1} = -\theta_{L+r,1}$, $r \leq L$), the eigenvalue is equal to

$$E_{lin} = 2[\sum_{r=1}^{L}\sum_{k=1}^{M}\arctan(2\pi(\lambda_k - \theta_{r,1})) - \sum_{p=L+1}^{2L}\sum_{k=1}^{M}\arctan(2\pi(\lambda_k - \theta_{p,1}))], \tag{4}$$

where λ_k are the solutions of the same set of the Bethe ansatz equations (2), because these two operators (lattice and linearized Hamiltonians) are functions of the same transfer matrix Eq. (1), and, thus, they commute. This yields immediately, that the lowest excitation energy

$$E_{lin}^{exc} = \sum_{r=1}^{L}\arctan[\frac{\cosh(\pi\lambda_0)}{\sinh(\pi\theta_{r,1})}] ,$$

where λ_0 is the excitation's rapidity, has a gap. It is not an even function of $\theta_{r,1}$ which characterize the spin chirality of the system, see [11], so this excitation carries nonzero chirality or topological charge. That is why we can say that such low-lying excitations could form the chiral spin liquid state [3].

Comparing the results for the lattice and linearized version of the multichain model we can conclude that the order of taking the long-wave length limit and linearization near Fermi points and solving of the Schrödinger problem (or the definition of the Hamiltonian) plays dramatic role. Two exactly solvable versions (the lattice and linearized long-wave ones) of the multichain quantum antiferromagnet have qualitatively different answers for the spin gap of elementary excitations, despite the fact that the transfer matrix and the Bethe ansatz equations are the same for each model. The reason for such dramatic difference is due to the noncommutativity of the linearization near Fermi points and taking the thermodynamic limit ($N, M \to \infty$, M/N fixed), which is usual for many-body problems. It means that the order of limitation for the size of a system and for the characteristic wavelength for the spin excitation plays an essential role here.

The results obtained in this note are easily generalized for the spin S multichain problem: the difference is only because the ground state is organized as the Dirac sea filling for string length $2S$ solutions of Eqs. (2) (we must replace $f(2x)$ in rhs of Eqs. (2) with $f(x/S)$ and we have to remind that strings of any length are fermions, too, because only no more then one string may exist with one λ, [6]). But the main result holds unchanged: for the lattice Hamiltonian the low-lying excitation (spinon) is gapless, but for its continuum linearized near Fermi points version the lowest excitation has a gap. The same result holds for any exactly solvable strongly correlated electron model (i.e., for a system with internal degrees of freedom), for which the Hamiltonian for one chain is the generalized permutation operator (see, e.g., the exactly solvable multichain supersymmetric $t-J$ model [17]).

In summary, we have argued, solving exactly the eigenfunction and eigenvalue problems for the multichain quantum antiferromagnet with T and P symmetries violation, that the lattice version of the model has no gap in the spectrum of low-lying excitations, but the eigenvalue of the linearized near Fermi points Hamiltonian of the model with the same transfer matrix has a gap. The result could be of use for correct quantum description of low-lying excitations not only for multichain spin models like ladder systems [12], but, also, for lattice $(1+1)$ field theory models, and for 1D quantum spin S Heisenberg multisublattice antiferromagnets.

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