

Nonlinear Dynamics of Surface Dust Vortex and Dust Zonal Flow Systems

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We present analytical and simulation studies of highly resolved dust fluid flows involving nonlinearly coupled incompressible surface dust vortex modes (SDVMs) and dust zonal flows (DZFs) in *nonuniform* unmagnetized dusty plasmas. For this purpose, we use the hydrodynamic equations for the dust fluid and Boltzmann distributed electrons and ions, and obtain a set of equations which exhibit nonlinear couplings between the SDVMs and DZFs. The nonlinear equations are then used to investigate the parametric excitation of DZFs by the Reynolds stresses of the SDVMs. Large scale SDVMs emerge through nonlinear interactions with DZFs, and they suppress the dust particle transport across the density gradient. By contrast, DZFs possess short scale vortices with a higher turbulent transport. The relevance of our investigation to the role of coherent structures in a nonuniform dusty plasma is discussed.

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Dust and dusty plasmas are ubiquitous in space and laboratory discharges [1–3]. In space, dusty plasmas can be found in accretion disks, supernova remnants, interstellar clouds, planetary magnetospheres, cometary tails, and in the Earth's ionosphere/mesosphere. In laboratory discharges, dusty plasmas play a role in plasma processing, edge plasmas in magnetic fusion devices, and in microelectronics fabrication. Dusty plasmas are composed of electrons, ions, and charged micro dust particles. In most space and laboratory dusty plasmas, dust grains are weakly correlated since the coupling parameter Γ (the ratio between average Coulomb interaction energy and the average particle thermal energy) is much smaller than 1. On the other hand, in dusty plasmas with $\Gamma \geq 1$, one encounters dust Coulomb crystals [4], dust micro-bubbles [5], and dust Coulomb balls [6], which are a manifestation of novel collective processes in dusty plasmas. Dusty plasmas exist in “liquid” and “crystalline phases”, as well as in the “gaseous” state [7]. While enormous work has been carried out to understand the nonlinear dynamics of this complex plasma state, there exist a number of outstanding issues and largely unexplored nonlinear dynamics that need complementary approaches for a broader universal understanding. With the advent of sophisticated controlled laboratory experiments and computer simulations [2, 8, 9], we have advanced our knowledge of nonlinear effects in complex (dusty) plasmas. For instance, recent ex-

perimental investigations [8, 9] of dust “nanofluidics” show the formation of dust shear flows at kinetic levels and also measure the dust viscosity. Nevertheless these experiments, studying the properties of dust fluid turbulence on a kinetic level, require the knowledge of energy transfer due to nonlinear mode couplings associated with harmonic generation, which is considered as a generic feature of a fully developed plasma and fluid turbulence. Moreover, sheared flows and nonlinear structures are quite common in hydrodynamics [10] and Martian atmospheres [11] though, their self-consistent evolution in a two-dimensional (2D) Yukawa system has not been explored as yet. Our present Letter, therefore, investigates through fully self-consistent nonlinear fluid simulations the formation of long-scale flows due to surface dust vortex modes (SDVMs) and dust zonal flows (DZFs) in nonuniform unmagnetized, dissipative dusty plasmas. Interestingly, we find that the long-scale flows in our work are excited across a nonuniform dust density gradient; a situation encountered most frequently in numerous laboratory experiments and space plasmas.

Our investigations are based on a model put forward by Hasegawa and Shukla [12], who theoretically pointed out the existence of incompressible SDVMs in a nonuniform, unmagnetized dusty plasma. We first discuss the equilibrium state of our partially ionized laboratory dusty discharges in which collisions between stationary neutrals with electrons and ions are more frequent than those between electrons and ions. In the dust particle loaded discharge, one obtains [13] from the

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conservation of the electron and ion fluxes the ambipolar electric field $E_{0z} = D_i \chi^{-1} \partial n_{i0} / \partial z - D_e \chi^{-1} \partial n_{e0} / \partial z$, where $D_j = T_j / m_j \nu_{jn}$ and $\mu_{jn} = e / m_j \nu_{jn}$ are the diffusion and mobility of the particle species j (j equals e for electrons, i for ions), e is the magnitude of the electron charge, T_j is the temperature, m_j is the mass, ν_{jn} is the rate of the electron and ion-neutral collisions, $\chi = \mu_i n_{i0} + \mu_e n_{e0}$, $n_{i0} = n_{e0} + Z_d n_{d0}$, n_{i0} , n_{e0} , n_{d0} are the unperturbed number densities of the ions, electrons, and dust grains, respectively, and Z_d is the number of the electrons on a dust grain. The vertically upward (along the z axis) ambipolar electric field E_{0z} can levitate a negatively charged dust particle due to a balance between the electric force ($= -Z_d e E_{0z}$) and the vertically downward gravity force $-m_d g$, where m_d is the dust mass, and g is the gravity constant. Nitter [14] has also discussed the levitation of a charged dust in the plasma sheath of rf and dc glow discharges where the electric field E_{0z} is obtained by solving Poisson's equation together with a Boltzmann electron density distribution and the ion density distribution deduced from the steady state continuity and momentum equations for collisional ions.

When the equilibrium is perturbed, one has the possibility of nonlinear SDVMs in the form of a dipolar vortex [15] or a chain of vortex [16], which can be associated with coherent vortical structures in laboratory experiments [17–21]. The concept of drift wave driven zonal flows in a collisional electron-ion plasma (without dust) was introduced by Hasegawa and Wakatani [22]. When the phase speed (wavelength) of the SDVMs and DZFs is much smaller than the electron and ion thermal speeds (electron and ion collisional mean free paths $V_{Te, Ti} / \nu_{en, in}$, where $V_{Te} (V_{Ti})$ is the electron (ion) thermal speed in dusty plasmas, the perturbed electrostatic forces ($q_j n_{e0, i0} \mathbf{E}_1$) acting on electrons and ions balance the corresponding pressure gradient $-\nabla P_{j1}$, where $q_e = -e$, $q_i = e$, and \mathbf{E}_1 (P_{j1}) is the perturbed electric field (perturbed pressure). The dynamics of incompressible ($\nabla \cdot \mathbf{v}_d = 0$) SDVMs and DZFs is then governed by the dust continuity and dust momentum equations, namely [12]

$$\partial \rho_d / \partial t + \nabla \cdot (\rho_d \mathbf{v}_d) = 0, \quad (1)$$

and

$$\rho_d (\partial / \partial t + \nu_d - \eta \nabla^2 + \mathbf{v}_d \cdot \nabla) \mathbf{v}_d = -\nabla P_1 + \rho_d g \hat{\mathbf{z}}, \quad (2)$$

where $\rho_d = m_d (n_{d0} + n_{d1})$, $n_{d1} (\ll n_{d0})$ is a small perturbation in the equilibrium dust number density, \mathbf{v}_d is the perturbed dust fluid velocity, ν_d is the dust-neutral collision frequency, η represents the kinematic

dust fluid viscosity (typically $\simeq 10^{-2} - 10^{-1} \text{ cm}^2/\text{s}$ in laboratory, similar to that of water $\sim 10^{-2} \text{ cm}^2/\text{s}$), and $P_1 = P_{e1} + P_{i1} + P_{d1}$ is the perturbation in the equilibrium pressure. We stress that Eq. (2), in which the electric force on charged dust grains is eliminated by using $\mathbf{E}_1 = -(e/Z_d n_d) \nabla (P_{e1} + P_{i1})$, is widely used in the investigation of collective processes in dusty plasmas [15]. Furthermore, the effect of dust charge fluctuations can be neglected [15] since the timescale of our interest is longer than the dust charging period (typically microseconds for dusty plasma discharges).

Two-dimensional incompressible SDVMs and DZFs are characterized by the velocity vectors $\mathbf{v}_s = \hat{\mathbf{x}} \times \nabla \psi_s(y, z)$ and $\mathbf{v}_z = \hat{\mathbf{x}} \times \nabla \psi_z(y, z)$, respectively. Here, $\hat{\mathbf{x}}$ is the unit vector along the x axis, which is perpendicular to the z axis, and ψ_s and ψ_z are the stream functions of the SDVMs and DZFs, respectively. Thus, there exist SDVM and DZF vorticities characterized by $\Omega_s = \nabla \times \mathbf{v}_s \equiv \hat{\mathbf{x}} \nabla_\perp^2 \psi_s(y, z)$ and $\Omega_z = \nabla \times \mathbf{v}_z \equiv \hat{\mathbf{x}} \nabla_\perp^2 \psi_z$. Letting $\rho_d = \rho_{d0} + \rho_{ds}$ and $\mathbf{v}_d = \mathbf{v}_s + \mathbf{v}_z$ in Eqs. (1) and (2), we obtain the governing equations for the SDVMs in the presence of DZFs. We have

$$\left(\frac{\partial}{\partial t} + \hat{\mathbf{x}} \times \nabla \psi_s \cdot \nabla \right) \rho_{ds} + \rho'_{d0} \frac{\partial \psi_s}{\partial y} + (\hat{\mathbf{x}} \times \nabla \psi_z \cdot \nabla) \rho_{ds} = 0, \quad (3)$$

and

$$\left(\frac{\partial}{\partial t} + \nu_d - \eta \nabla^2 + \hat{\mathbf{x}} \times \nabla \psi_s \cdot \nabla \right) \nabla_\perp^2 \psi_s + g \frac{\partial \rho_{ds}}{\partial y} + (\hat{\mathbf{x}} \times \nabla \psi_s \cdot \nabla) \nabla_\perp^2 \psi_z + (\hat{\mathbf{x}} \times \nabla \psi_z \cdot \nabla) \nabla_\perp^2 \psi_s = 0. \quad (4)$$

where $\rho'_{d0} = \partial \rho_{d0} / \partial z$ and $\rho_{d0}(z) \ll \rho_{ds}$. We note that $\rho_{dz} \ll \rho_{ds}$ due to insignificant variation of the dust zonal flows (ZF) stream function along the y axis.

The dynamics of DZFs in the presence of the SDVMs is governed by

$$\left(\frac{\partial}{\partial t} + \nu_d - \eta \nabla^2 + \hat{\mathbf{x}} \times \nabla \psi_z \cdot \nabla \right) \nabla_\perp^2 \psi_z + \Omega_{\text{ave}} = 0, \quad (5)$$

where $\Omega_{\text{ave}} = \langle \hat{\mathbf{x}} \times \nabla \psi_s \cdot \nabla \nabla_\perp^2 \psi_s \rangle$ and the angular bracket denotes the averaging over the SDVM period. In the absence of the nonlinear interactions, the SDVMs and DZFs are decoupled. The corresponding dispersion relations, obtained from (3)–(5), are $\omega^2 + i\omega(\nu_d + \eta k_\perp^2) - \Omega_B^2 k_y^2 / (k_y^2 + k_z^2) = 0$ and $\Omega + i(\nu_d + \eta k_\perp^2) = 0$, respectively. Here, $\omega(\Omega)$ is the frequency of the SDVMs (DZFs), k_y (k_z) is the component of the wavevector along the y (z) axis. The buoyancy frequency squared is denoted by $\Omega_B^2 = g \partial \ln \rho_{d0} / \partial z$. If the equilibrium dust density is proportional to $\exp(-z/L)$,

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Fig.1. Time evolution of the SDVMs yields long scale potential flows (left) for which k_z is finite and $k_y \approx 0$. The vorticity component $\nabla^2 \psi_s$ consequently possesses elongated small scale structures

where $L = \rho_{d0}/\rho'_{d0}$ is the dust density gradient scale-size, then $\Omega_B^2 = -g/L$. We note that the latter is a positive definite for $L < 0$. In the absence of dissipation, we observe the frequency condensation of SDVMs for $k_z \gg k_y \rightarrow 0$, indicating the possibility of SDVMs driven DZFs with short scale structures along the z axis, as discussed below.

We first consider the parametric excitation of DZFs by large amplitude SDVMs. For this purpose, we neglect the self-interaction mode couplings in (3)–(5) and let $\rho_{ds} = \rho_{0\pm} \exp(\pm i \mathbf{k}_0 \cdot \mathbf{r} \mp i \omega_0 t) + \sum_{+,-} \rho_{\pm} \exp(i \mathbf{k}_{\pm} \cdot \mathbf{r} - i \omega_{\pm} t)$, $\psi_s = \psi_{0\pm} \exp(\pm i \mathbf{k}_0 \cdot \mathbf{r} \mp i \omega_0 t) + \sum_{+,-} \psi_{\pm} \exp(i \mathbf{k}_{\pm} \cdot \mathbf{r} - i \omega_{\pm} t)$, and $\psi_z = \varphi \exp(i \mathbf{q} \cdot \mathbf{r} - i \Omega t)$, where the subscript $0\pm$ (\pm) stands for the SDVM pump (sidebands), $\mathbf{k}_{\pm} = \mathbf{q} \pm \mathbf{k}_0$, and $\omega_{\pm} = \Omega \pm \omega_0$. Fourier transforming (3)–(5) and matching the phases, we obtain

$$H_{\pm} \psi_{\pm} = \pm i \frac{\omega_0 \hat{\mathbf{x}} \times \mathbf{q} \cdot \mathbf{k}_0}{k_{\pm}^2} (g k_{0y} + 2k_{\perp 0}^2 - q_{\perp}^2) \psi_{0\pm} \varphi, \quad (6)$$

and

$$(\Omega + i\nu) \varphi = i \frac{\hat{\mathbf{x}} \times \mathbf{k}_0 \cdot \mathbf{q}}{q_{\perp}^2} (K_{-}^2 \psi_{0+} \psi_{-} - K_{+}^2 \psi_{0-} \psi_{+}), \quad (7)$$

where $H_{\pm} = \omega_{\pm}^2 + i(\nu_d + \eta k_{\pm}^2) \omega_{\pm} - \Omega_B^2 k_{0y}^2 / k_{\pm}^2$, $k_{\pm}^2 = k_{0y}^2 + k_{z\pm}^2$, $K_{\pm}^2 = k_{\perp \pm}^2 - k_{0\perp}^2$, and $\nu = \nu_d + \eta q_{\perp}^2$. Eliminating ψ_{\pm} from (7) by using (6), we have the nonlinear dispersion relation

$$\Omega + i\nu = \omega_0 \frac{|\hat{\mathbf{x}} \times \mathbf{k}_0 \cdot \mathbf{q}|^2}{q_{\perp}^2} \times (g k_{0y} + 2k_{\perp 0}^2 - q_{\perp}^2) \sum_{+,-} \frac{K_{\pm}^2 |\psi_0|^2}{H_{\pm}}, \quad (8)$$

where $|\psi_0|^2 = \psi_{0+} \psi_{0-}$. For $q_{\perp} \ll |\mathbf{k}_0|$ we obtain from (8) $\Omega(\Omega + i\nu) = (|\hat{\mathbf{x}} \times \mathbf{k}_0 \cdot \mathbf{q}| / q_{\perp}^2) (g k_{0y} +$

$2k_{\perp 0}^2) \mathbf{q} \cdot \mathbf{k}_{0\perp} |\psi_0|^2$, which predicts a purely growing instability ($\Omega = i\gamma$) if the growth rate $\gamma > \nu$ and $(g k_{0y} + 2k_{\perp 0}^2) \mathbf{q} \cdot \mathbf{k}_{0\perp} < 0$. For some typical values, viz $|\mathbf{v}_s| \sim C_D$, $|\mathbf{k}/\mathbf{k}_0| \sim 0.1$, we have $\gamma \sim$ one tenth of the dust acoustic wave frequency $k_0 C_D$, where C_D is the dust acoustic speed [15].

Next, in order to study the dynamics of nonlinearly coupled SDVMs and DZFs, we develop a nonlinear code to carry out high resolution computer simulations of (3)–(5) in a periodic box of length π in each directions. The time integration uses a second order predictor-corrector method. The spatial resolution is 1024×1024 Fourier modes. All fluctuations in the simulations are initialized with a Gaussian random number generator to ensure that the Fourier modes are all spatially uncorrelated and randomly phased. This ensures a nearly isotropic initial condition in the real space. We further make sure that no asymmetry is introduced in the dynamical evolution by the initial spectra and the boundary conditions. The normalization is as follows; ρ_{ds} by ρ_{d0} , $\psi_{s,z}$ by $C_D \lambda_D$, t by ω_{pd}^{-1} , and the space variable by $\lambda_D = C_D / \omega_{pd}$, where $\lambda_D = C_D / \omega_{pd}$ is the dusty plasma Debye radius [15]. Thus, the free parameters in our simulations are $\nu_d / \omega_{pd} \sim 10^{-2} - 10^{-3}$, $\eta / C_D \lambda_D \sim 10^{-2}$, $\lambda / L \sim 0.01$ and $g \lambda / C_D^2 \sim \lambda_D / L_p \sim 0.01$, where $L_p^{-1} = -P_0^{-1} \partial P_0 / \partial z$. In Figs.1 and 2 we show the evolutions of the stream functions and vorticities of the SDVMs and DZFs, respectively. We see that stream functions are coherent, contrary to the irregular vorticities. This is an indication of the dual cascade, in which energy from short scale SDVMs is transferred to large scale DZFs. It is pertinent to note from Eqs. (3)–(5) that the mode $k_y \approx 0$ cannot be excited in a linear regime in which this mode is entirely absent from the dynamics.

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Fig.2. The dust zonal flows (DZF) tends to cascade towards shorter length scales that eventually align along the flow direction

Nevertheless, the $k_y \approx 0$ mode, essentially leading to an asymmetric large scale flows in a real space, is generated purely as a result of nonlinear interactions among SDVMs and DZFs in an inertial range spectral space across a large equilibrium dust density gradient. More precisely, in the SDVMs, the energy flows towards smaller k 's, so that we expect large scale structures, similar to the modified Navier–Stokes (NS) turbulence. On the other hand, DZFs are zero-frequency limit of the SDVM mode; they have very small k_y and very large k_z , so that the frequency condensation occurs for short wavelengths (viz. $k_z \gg k_y$) structures. DZFs are excited due to the Reynolds stresses of the SDVMs. While the Reynolds stresses possess a tendency of typically generating the large-scale flows, their time average appearing in Eq. (5) causes a net nonlinear dissipation of zonal flows. This appears to be the primary reason why DZFs form short scales (see Fig.2). The vorticity field of DZFs in Fig.2 appears to be stretched across the equilibrium density gradient wrapping the small scale dust vortex structures around in the nonlinear saturated state. The vortices are trapped in the horizontal sheared flow and propagate along the self-consistent flow across the equilibrium gradient. The energy spectrum decays due to dust-neutral collisions and dust kinematic viscosity at smaller scales. Figure 3 exhibits a high resolution Kolmogorov-like spectrum of a fully developed coupled SDVM-ZF turbulent system. The spectrum of SDVMs is evidently steeper than that of DZFs in the inertial regime, thereby indicating the presence of large scale structures in its spectrum. This is further consistent with Fig.1, which demonstrates $k_y \approx 0$ flows in the saturated SDVM-ZF turbulent state. It is to be noted that both the spectra in Fig.3 are steeper compared to the fully developed 2D

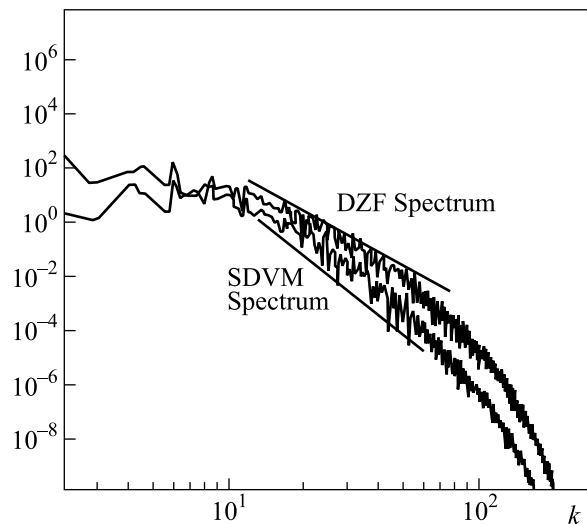


Fig.3. Kolmogorov-like turbulent spectra of coupled DZF and SDVMs system. The spectrum of SDVMs in inertial range turbulence is more steep than that of DZF due to the presence of large scale flows that have $k_y \approx 0$. The numerical resolution is 1024×1024 Fourier modes in a two-dimensional box

turbulent spectra for enstrophy or energy due to large-scale structures that condensate the lower Fourier modes because of inverse cascade processes. The dust density fluctuations also cascade towards long-scale structures due to short scale vortex merging. There also exist non-thermal transport associated with the effective turbulent diffusion ($D_{\text{eff}} = \int_0^\tau d\tau \langle v(y, t=0)v(y, t+\tau) \rangle$) of a dust particle in large scale ZF structures due to a random walk of the macroparticles in enhanced zonal flow fluctuations. As expected, emergence of large-scale coherent flows in SDVMs quench turbulent transport, in

contrast to that involving random short scale DZFs, as shown in Fig.4. In nonuniform, nonlinear media without dust, 2D flows have been explained on the basis of the NS and the Charney-Hasegawa-Mima (CHM) equations [23, 24], which have two constants of motion, namely the energy and enstrophy (squared vorticity). The energy decays from a source to long wavelength, while enstrophy flows to shorter scales. Such a dual cascading, in agreement with the statistical quasi-equilibrium theory, ensures the formation of coherent vortical structures (eddies) [25], which are responsible for producing enhanced transport of fluids and plasma particles. For unbounded 2D NS turbulence, the conserved quantities are the kinetic energy $E = \int_0^\infty E(k)dk$ and fluid enstrophy $Z = \int_0^\infty k^2 E(k)dk$, where E_k is the kinetic energy spectrum. For CHM turbulence, the conserved quantities are the total energy $E + \lambda^2 I$ and total enstrophy $Z + \lambda^2 E$, where $I = \int_0^\infty k^{-2} E(k)dk$ and λ is a positive constant.

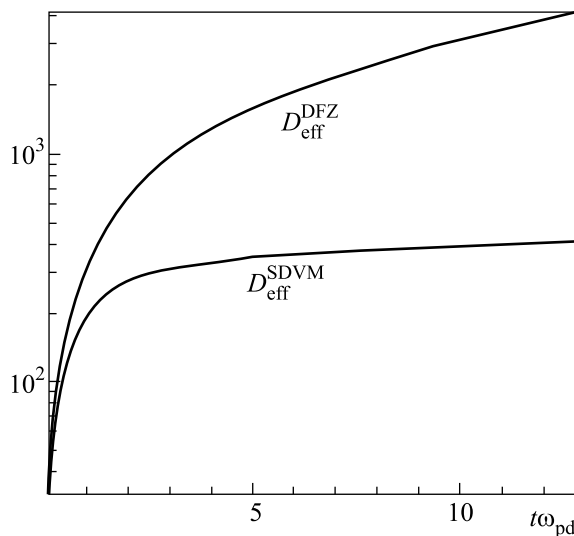


Fig.4. The effective diffusion coefficient associated with nonlinear turbulent transport in a coupled SDVM-DZF system. In agreement with Fig.3, transport is suppressed due to the presence of large-scale flows in SDVM, while it has enhanced in short scales DZFs

To summarize, we have presented a nonlinear mechanism by which DZFs can be generated on account of the energy of incompressible SDVMs in a nonuniform unmagnetized dusty plasma. Specifically, we have presented a system of equations which shows that nonlinear couplings between the SDVMs and DZFs occur due to interactions between the SDVM density fluctuation and the velocity fluctuation of DZFs, as well as due to the dust fluid advection in the coupled SDVM-DZF system. The coupled mode equations are analyzed to show the existence of the modulational instability, which drives

DZFs at nonthermal levels. The dynamics of nonlinearly interacting SDVMs and DZFs reveals interesting features of dual cascading leading to the formation of large scale dust vortices. The latter can produce dust particle transport across the density gradient. In fact, localized dust grain structures in association with dust grain transport have been observed in nonuniform laboratory dusty plasma discharges [17–21].

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