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# SELF ENERGY CONTRIBUTION TO THE GROUND STATE HUPERFINE SPLITTING OF $Bi^{82+}$

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The self energy contribution to the hyperfine splitting of the ground state of $Bi^{82+}$ is calculated for the point nucleus. It is found that the theoretical value of the wavelength of the ground state hyperfine splitting transition of $^{209}Bi^{82+}$ constitutes $\lambda = 244(1)$ nm and is in good agreement with experiment.

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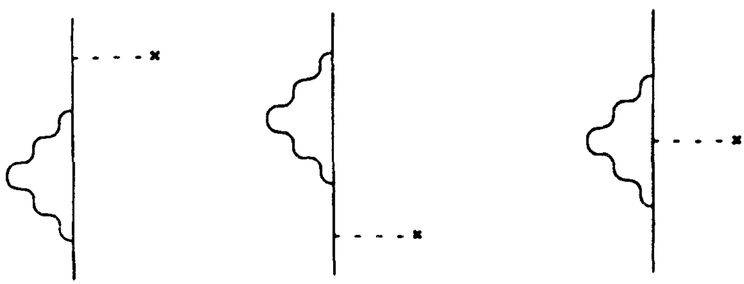
Recently [1] the experimental value of the wavelength of the ground state hyperfine splitting transition of $^{209}Bi^{82+}$ was found to be $\lambda = 243.87(4)$ nm. The related theoretical value calculated for $\mu = 4.1106(2)\mu_N$ [2] without taking account of the QED correction is $\lambda = 242.0$ nm [3-5] (the value $\lambda = 242.3$ nm found in [4] was reduced by 0.3 nm in [5] by taking into account the angular asymmetry of the spin distribution of the nuclear moment). The uncertainty of this value is mainly defined by deviation from the single particle approximation used in the calculation of the Bohr–Weisskopf correction. The QED correction consists of the self energy (SE) and vacuum polarization (VP) contributions. The VP contribution was evaluated in [6] within the Uehling approximation to give a $\Delta\lambda = -1.6$ nm shift. The present paper is devoted to the calculation of the SE contribution for the point nucleus. To calculate this contribution we use the method that was developed in our recent paper [7] for calculation of the self energy screening diagrams.



a b

Self energy-hyperfine interaction diagrams

The self energy contribution to the hyperfine splitting is defined by the diagrams shown in Figure where the dotted line denotes the hyperfine interaction. The calculation expressions for these diagrams can be easily derived by the two-time Green function method [8]. The diagrams in Figure *a* are conveniently divided

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into irreducible and reducible parts. The reducible part is the part in which the intermediate state energy (between the self energy and the hyperfine interaction line) coincides with the initial state energy. The irreducible part is the remaining one. The irreducible part ($\Delta E_{irred}$) is calculated in the same way as the first order self energy contribution. For a point nucleus considered here the external wave function containing the hyperfine interaction line is calculated analytically by using the generalized virial relations for the Dirac–Coulomb problem [9, 10] ( the sign at $\varepsilon_{n\kappa}$ in the second line of the equation (18) in [10] must be changed; all calculations in [10] were carried out with the correct sign). The reducible contribution ($\Delta E_{red}$) is grouped with the vertex contribution ($\Delta E_{ver}$) (Figure *b*) to be calculated without renormalization. The sum of these contributions is conveniently divided into three parts

$$\Delta E_{red} + \Delta E_{vertex} = \Delta E_{pole} + \Delta E_{vr} + \Delta E_{infr} \,. \tag{1}$$

Here $\Delta E_{pole}$ is the part of the vertex contribution with $\varepsilon_{n_1} = \varepsilon_a$ and $\varepsilon_{n_2} \neq \varepsilon_a$ and vice versa where $a$ is the initial ($1s$) electron state and $n_1$, $n_2$ are the intermediate electron states. $\Delta E_{infr}$ is the non-vanishing contribution from the sum of the infrared divergent vertex term ($\varepsilon_{n_1} = \varepsilon_{n_2} = \varepsilon_a$) and the infrared divergent reducible term ($\varepsilon_n = \varepsilon_a$). $\Delta E_{vr}$ is the sum of the reducible and vertex contributions with the pole and infrared terms subtracted. The term $\Delta E_{pole}$ is finite. We calculated it by using the B spline method for the Dirac equation [11]. To avoid a numerical calculation of matrix elements of the too singular ($\sim r^{-2}$) hyperfine interaction operator with the Dirac–Coulomb wavefunctions, these matrix elements were expressed in terms of less singular ($\sim r^{-1}$) operators by using the generalized virial relations for the Dirac equation [9]. The infrared term $\Delta E_{infr}$ is finite and its calculation caused no problems. To calculate $\Delta E_{vr}$ the reducible and vertex contributions were grouped by the partial wave expansion of the photon propagator. The complete result of the calculation is conveniently written in the form

$$\Delta E^{SE}_{hfs} = -3.8\,\alpha E^{nr}_{hfs} \,, \tag{2}$$

where $\alpha$ is the fine structure constant, $E^{nr}_{hfs}$ is the non-relativistic value of the ground state hyperfine splitting. This contribution gives a $\Delta\lambda = 3.6\,\mathrm{nm}$ shift for the transition wavelength. Adding the SE shift and the VP shift (-1.6 nm) calculated within the Uehling approximation in [6] to the value $\lambda = 242.0$ nm we obtain $\lambda = 244.0$ nm that is in good agreement with the experimental value $\lambda = 243.87(4)$ nm. However, we estimate that an uncertainty of the theoretical value caused by the nuclear magnetization and charge distributions is about $\pm$ 1.0 nm.

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