

## ON THE DECONFINEMENT PHASE TRANSITION IN THE RESONANCE GAS

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We obtain the constraints on the ruling parameters of the dense hadronic gas model at the critical temperature and propose the quasiuniversal ratios of the thermodynamic quantities. The possible appearance of thermodynamical instability is discussed.

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1. The thermodynamics of a dense hadron gas is bound to be a focal point of the analysis of possible phase transformations in the hadronic matter and their manifestations. With the energies of the heavy ion collisions available at present and, most importantly, in the near future the topic is no longer a purely theoretical one. In high energy nuclear collisions we observe the thermodynamics of strong interactions in action [1]. If we imagine the initially dilute hadronic gas being heated, then at some point the hadronic excitations are overlapping to an extent that does not any longer permit to use the picture of a weakly interacting hadron gas. In this context the most reliable estimate of a temperature at which the hadron gas density is high enough and this description is no longer true is provided by a chiral theory [2] and gives a value of  $T' = 130$  MeV. At temperatures  $T > m_\pi$  we have a strong interacting dense hadronic system. As we believe that at high enough temperatures the true excitations are gluons and quarks, the crucial questions are at which temperature and how the hadron phase is transforming into a quark-gluon one. Indeed, it is highly improbable that the transition takes place within a weakly coupled hadronic regime (i.e. at  $T < T'$ ), and we are necessarily facing a problem of describing a strong coupled regime of a dense hot hadron gas. An ingenious way of dealing with this obviously desperate situation was proposed by Hagedorn (see, e.g., [3]). The idea was to rewrite a partition function of a strongly interacting system as that of an infinite number of resonances having zero width and characterized by a rapidly (exponentially) growing spectrum thus diagonalizing the initial hamiltonian<sup>2)</sup>. The resulting hadronic mass spectrum takes

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<sup>2)</sup>This idea was later successfully realized for two-dimensional conformal theories and integrable deformations of them (see, e.g., [4]).

a form

$$\tau(m) = c \frac{1}{m^a} \exp\left(\frac{m}{T_0}\right) \quad (1)$$

depending on three parameters  $a, c$  and  $T_0$ . The typical expression for the thermodynamic quantity (here at the example of a pressure of a relativistic gas of massive particles) has a form

$$P = \sum_i \frac{m_i^2 T^2}{2\pi^2} K_2\left(\frac{m_i}{T}\right) + \frac{T^2}{2\pi^2} \int_M^\infty dm \tau(m) m^3 K_2\left(\frac{m}{T}\right), \quad (2)$$

where the sum is taken over the discrete spectrum ( $m_i < M$ ) and the integration is performed over the continuous one. We emphasise that the fourth parameter has appeared: the cutoff mass  $M$ . Below we argue that it can in fact play a decisive role in the description of the hadronic phase.

2. Now when the framework of the discussion and the set of parameters of the hadronic phase is settled, let us briefly remind the existing possibilities for arranging the transition between the hadronic matter and the quark-gluon one. The key parameter is a power in the preexponential mass factor in Eq.(1) that determines the type of a possible singularity of a partition function. In particular, for  $a \leq 7/2$  the energy density becomes infinite at  $T = T_0$ , when  $a = 9/2$  the energy density is finite at  $T = T_0$ , but the specific heat has a logarithmic singularity similar to that in the second order phase transitions, and at  $a > 9/2$  the singularity moves to higher orders in the derivatives of the thermodynamic potentials (see, e.g., [3,5,6]). A more sophisticated treatment of hadron bag gas within alternative pressure ensemble leads to similar conclusions [7].

Now let us discuss at which temperature  $T_c$  the transition to the quark-gluon phase takes place. The physical situation will obviously depend on the value of the parameter  $a$ . Before listing the arising possibilities let us remind that the calculation of the thermodynamical quantities using the exponential spectrum (1) is far from being trivial. In a seminal paper [8] Carlitz has shown that for  $a > 5/2$  the equivalence of the canonical and microcanonical ensembles is no longer guaranteed and the system can be thermodynamically unstable (negative specific heat) thus making the statistical approach questionable. Below we give more comments on that.

In order to get a quantitative estimate of the possible value of the transition temperature  $T_c$  it is customary to equate the expressions for the pressure in the hadronic phase (with some given value of  $a$ ) to that in the quark-gluon gas, for which one usually takes the bag-motivated expression

$$P_{QGP}(T) = \frac{37\pi^2}{90} T^4 - B_0, \quad (3)$$

where  $B_0$  is a nonperturbative vacuum pressure (bag constant).

At  $a \leq 7/2$ , when  $T_0$  is a limiting temperature, there is no crossover of the pressure curves (2) and (3) (see, e.g., [9]). This has led the authors of several papers [10] to the necessity of introducing additional mass-dependent interactions eliminating the limiting temperature. However, this proposal contradicts the original idea, namely that of hiding all the interaction effects in the infinite spectrum of free resonances. Therefore it is interesting to look more closely at other possibilities of arranging a phase transition working only with a free spectrum (1).

3. The obvious attempt to handle this unfortunate situation is to try to construct a phase transition choosing  $a > 9/2$ , so that the thermodynamic potentials and their derivatives up to the second order (including the specific heat) are finite at  $T = T_0$ .

Before turning to the pressure equality equations let us stress again [8], that for  $a > 9/2$  we find ourselves in a domain where the thermodynamical stability of a system can break down. In particular, the specific heat is positive for the energy densities

$$\epsilon < \epsilon_{crit} = \frac{M}{V}, \quad (4)$$

where  $V$  is a volume of a system and  $M$  is a cut-off used in integrating over the continuous spectrum (see Eq. (2)). Let us now imagine the evolution of the quark-gluon matter created in heavy ion collisions or in the early Universe. If

$$\epsilon_{QGP}(T_c) > \epsilon_{crit}$$

the produced hadron matter will form a thermodynamically unstable system with the preferable configuration consisting of the particles all having masses of order of  $M$ . This could lead to interesting predictions for the heavy ion collisions. Namely that can give rise to a explosive production of particles with the masses of order of cut-off one. Here we would like to stress the crucial role of the cut-off mass  $M$ . The value of this cut-off is usually believed to be a matter of choice, because there is no natural border between the discrete and continuum spectrum. However as there are good grounds to believe that the masses in the discrete part of the spectrum (pions, nucleons, ...) significantly depend on the interaction of the corresponding particles (e.g., decrease with temperature), the account for interactions in the discrete spectrum forces is, generally speaking, to make corresponding changes in the value of the cut-off mass  $M$ . For example, if, following the "mean field" logic, the masses of the particles are decreasing with increasing density, thus forcing us to decrease the value of  $M$ , then because of the dropping  $\epsilon_{crit}$  we are increasing the probability of getting a thermodynamically unstable system. This is also potentially important for determining the temperature  $T'$ . The necessity of shifting the cut-off mass  $M$  towards lower values can invalidate the picture of a dilute hadron gas earlier than naively expected.

4. Let us now suppose, that  $\epsilon_{crit}$  is big enough so that the system is thermodynamically stable. Then one can carry out a standard Maxwell construction getting the intersection of the pressure curves for both phases and thus a phase transition point. This is of course an assumption, but at  $c_v > 0$  one can believe that the microcanonical and canonical ensembles are equivalent [11]. In the following we shall exploit a simplest hypothesis, namely that a transition takes place at  $T = T_0 = T_c$ . The densities of the thermodynamical quantities at  $T_c$  can be calculated analytically. Let us consider  $T_c < M$  (which holds for any realistic description of the phase transition in the hadronic matter). The contribution of the discrete spectrum at  $T_c$  is exponentially smaller than that of the continuum one and we get for the pressure and entropy

$$P(T_c) = \frac{cT_c^{5-a}}{(2\pi)^{3/2}} \sum_{n=0} \frac{(2, n)}{2^n} \frac{(T_c/M)^{a-5/2+n}}{a-5/2+n}, \quad (5)$$

$$S(T_c) = \frac{cT_c^{4-a}}{(2\pi)^{3/2}} \sum_{n=0} \frac{(3, n)}{2^n} \frac{(T_c/M)^{a-7/2+n}}{a-7/2+n}, \quad (6)$$

where  $(\nu, n)$  is a Hankel's symbol

$$(\nu, n) = \frac{\Gamma(1/2 + \nu + n)}{n! \Gamma(1/2 + \nu - n)}.$$

The conditions for the deconfinement transition to take place are (all quantities are taken in the critical point):

$$P_h = P_{QGP} > 0, \quad S_h < S_{QGP}, \quad (7)$$

where  $S_{h, QGP}$  are the entropy densities in the hadron and quark-gluon phases respectively. Working in the lowest order in the small parameter  $\rho = T_c/M$  and neglecting the contribution of the discrete spectrum we get the following restrictions on the values of the parameters present in the hadron and plasma partition functions:

$$\left(\frac{37\pi^2}{90}\right)^{1/4} \left(1 - 4\frac{1+\epsilon}{2+\epsilon}\rho\right)^{1/4} T_c < B_0^{1/4} < \left(\frac{37\pi^2}{90}\right)^{1/4} T_c \quad (8)$$

and

$$c < (2\pi)^{3/2} \frac{74\pi^2}{45} T_c^{7/2+\epsilon} (1+\epsilon)\rho^{1+\epsilon}, \quad (9)$$

where, as discussed before,  $\epsilon = a - 9/2 > 0$ . As the bag constant  $B_0$  should be positive, one also gets from Eq. (9):

$$\frac{T_c}{M} < \frac{2+\epsilon}{4(1+\epsilon)}. \quad (10)$$

The corresponding numerical restrictions on the possible values of the bag constant  $B_0$  and the parameter  $c$  for some typical values of the dimensionful parameters  $T_0$  and  $M$  and  $a = 5$  are given in the Table.

**Restrictions on model parameters at  $\epsilon = a - 9/2 = 0.5$**

$M$ (GeV)	$T_c$ (GeV)	$B_{0min}^{1/4}$ (GeV)	$B_{0max}^{1/4}$ (GeV)	$c_{max}$
1	0.16	0.20	0.23	0.02
1	0.2	0.24	0.28	0.05
1	0.24	0.27	0.34	0.15
2	0.16	0.21	0.23	0.01
2	0.2	0.26	0.28	0.02
2	0.24	0.31	0.34	0.05

Thus we see that the possibility of a phase transition heavily restricts the values of the parameters in the problem. This is especially clear for the values of the parameter  $c$  which are quite small. Let us notice however that phenomenologically there is practically no restrictions on the values of parameters in the spectrum (1), because this formula is supposed to be valid only for the highly excited resonances where the hadron spectrum is pretty unknown. In this respect the small allowed values of  $c$  are intuitively uncomfortable but not phenomenologically forbidden.

Let us note, that the structure of the expressions for the thermodynamical quantities Eqs. (5), (6) enables one to get the information about the phase

transition which is to a large extent independent of the parameters chosen to describe a hadronic phase near the first order phase transition point. The first example of such a quasiuniversal ratio is

$$\frac{P(T_c)}{E(T_c)} = \left(\frac{a - 7/2}{a - 5/2}\right) \frac{T_c}{M}, \quad (11)$$

where we are using the lowest order expressions in the small parameter  $T_c/M$ . As in order to describe a first order phase transition it is necessary to have  $a > 9/2$ , we get one more inequality for  $T_c/M$ :

$$\frac{1}{2} \frac{P(T_c)}{E(T_c)} < \frac{T_c}{M} < \frac{P(T_c)}{E(T_c)}. \quad (12)$$

Let us also calculate the derivative of the energy over temperature (as the calculations are done exclusively within the hadronic phase, the derivative should be understood as a left one). In the leading order in  $T_c/M$  we have

$$\left. \frac{\partial E}{\partial T} \right|_{T_c} = c T_c^{4-a} \frac{1}{a - 9/2} \left(\frac{T_c}{M}\right)^{a-9/2}. \quad (13)$$

Now we can form another interesting ratio

$$\frac{P(T_c) T_c E'(T_c)}{E^2(T_c)} = 1 + \frac{1}{(a - 9/2)^2 + 2(a - 9/2)}, \quad (14)$$

which for  $a > 9/2$  is always larger than one. Thus we see that the proposed description of the first order phase transition leads to some quasiuniversal ratios (weakly dependent only on the parameter  $a$  and independent of all other parameters) which are in some sense analogous to the universality in the description of the second order phase transition. In the picture we are discussing this is not surprising, because the exponentially growing spectrum leads to a powerlike temperature dependence in both situations, and the difference depends on the value of the parameter  $a$ . These restrictions seem to be fairly important for the calculations describing the matter evolution in high energy nuclear collisions and early Universe.

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