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FLUX NOISE NEAR THE BEREZINSKII – KOSTERLITZ – THOULESS TRANSITION

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We study the flux noise in Josephson junction arrays in the critical regime above the Berezinskii–Kosterlitz–Thouless transition. In proximity coupled arrays a local ohmic damping for the phases is relevant, giving rise to anomalous vortex diffusion and a dynamic scaling of the flux noise in the critical region. It shows a crossover from white to $1/f$-noise at a frequency $\omega_\xi \propto \xi^{-z}$ with a dynamic exponent $z = 2$.

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A variety of two dimensional systems undergo the Berezinskii – Kosterlitz – Thouless (BKT) pair-unbinding transition [1, 2]. The transition between the high temperature (disordered) and the low temperature (coherent) phase is driven by the thermally excited vortices. These topological excitations form a two dimensional Coulomb gas. Below the BKT transition temperature $T_{\rm BKT}$ they are bound in vortex-antivortex pairs. Above the critical temperature the pairs dissociate and form a vortex plasma. In the plasma phase the vortex-vortex interaction is screened at a distance of the order of the correlation length ξ which diverges at $T_{\rm BKT}$.

Arrays of Josephson junctions are prototype systems to study the BKT transition. Below the BKT transition the array is phase coherent and thus superconducting. On the contrary, in the plasma phase, free vortices destroy the coherence and the system is resistive, though each island remains superconducting. In the last decade there has been a great amount of work on the various aspects of the BKT transition in Josephson arrays (see Ref. [3] for an overview). Experimental studies are based on electrical resistance [4], two-coil inductance [5, 6], and superconducting quantum interference device (SQUID) [7, 8] measurements.

Anomalous suppression of the vortex mobility has been observed in proximity coupled Josephson junction arrays by Théron et al. [6]. The vortex mobility vanishes logarithmically at low energies, showing that vortex motion in these arrays *cannot* be described by a Drude model with a constant mobility. This behavior has been explained either introducing a local ohmic damping for the phases of the superconducting wave function [9, 10] or by invoking long-range vortex interactions [11]. More recently a regime interpretation of the vortex dynamics was given in Ref.[12].

In a recent Letter, Shaw et al. [7] investigated the magnetic flux noise close to the BKT transition in an overdamped Josephson junction array by means of a SQUID. The flux-noise spectrum is defined as

$$S_\Phi(\omega) = \int dt e^{i\omega t} \langle \Phi(t)\Phi(0) \rangle \,, \tag{1}$$

where $\Phi(t)$ is the flux detected by the SQUID and is proportional to the vortex density integrated over the SQUID area. Near the transition the flux noise is white for frequencies $\omega < \omega_\xi$ crossing over to a $1/f$-noise for $\omega > \omega_\xi$. The crossover frequency ω_ξ vanishes as the transition is approached from above as $\omega_\xi \propto \xi^{-z}$, where ξ is the correlation length and $z \approx 2$ is the dynamic exponent. All the data obtained at different temperatures collapse on a single scaling curve when plotted as a function of ω/ω_ξ. The problem was studied by numerical simulations [13, 14] of both the Time Dependent Ginzburg – Landau (TDGL) and the Resistive Shunted Junction (RSJ) models.

In this Letter we study theoretically the flux noise in the TDGL model near the BKT transition. We determine the scaling function of the flux noise analytically in the limit in which the SQUID size is much smaller than the correlation lenght.

In order to study the flux noise we map the XY-model with a local damping onto a 2D (dynamic) Coulomb gas [15]. We determine the dynamic correlation functions using an improved Debye-Hückel (DH) approach for the Coulomb gas in the same spirit as in Ref. [16].

The Euclidean action for the XY-model with local damping for the phases φ is

$$S[\varphi] = E_J \int_0^\beta d\tau \sum_{<ij>} [1 - \cos(\varphi_i(\tau) - \varphi_j(\tau))] +$$

$$+ \frac{\alpha}{8\beta^2} \int_0^\beta d\tau d\tau' \sum_i \left(\frac{\varphi_i(\tau) - \varphi_i(\tau')}{\sin[\pi(\tau - \tau')/\beta]} \right)^2 \,, \tag{2}$$

where E_J is the Josephson coupling between neighboring sites. The local ohmic damping introduces an interaction of the phase of a single grain at different times. In proximity coupled arrays, which consist of superconducting island on top of a metallic film, the local damping parameter α is related to the shunting resistance R of one island to the substrate, $\alpha = h/4e^2R$.

A Villain transformation [17] is used to derive an effective dynamic action for the vortices. Here we follow the method described in Ref. [18] and obtain the dynamic action for the vortex degrees of freedom $v_i(\tau)$

$$S[v] = \frac{1}{2} \int_0^\beta d\tau d\tau' \sum_{i,j} v_i(\tau) D_{ij}(\tau - \tau') v_j(\tau') \,. \tag{3}$$

The kernel D reads

$$D(k, \omega_\mu) = \frac{4\pi^2 E_J}{k^2} \frac{k^2 + 2|\omega_\mu|/\omega_\alpha}{k^2 + |\omega_\mu|/\omega_\alpha} \,, \tag{4}$$

with $\omega_\alpha = 2\pi E_J/\alpha$, and ω_μ are the Matsubara frequencies. We choose the lattice spacing as the unit of length scales. In the static case this yields the standard 2D Coulomb gas.

The evaluation of the flux noise is done by means of a two-step procedure in the spirit of Ref. [16]. i)The physics taking place at length scales shorter than ξ is taken into account by using the scaling properties of the flux noise. ii) At the scale of the order of the correlation lenght, all the dipoles have been integrated out and, since the physics above $T_{\rm BKT}$ is dominated by screening, a DH approximation (with renormalized parameters) is used [19, 16].

The effect of bound pairs up to distances of the order of the correlation length can be account for by means of the scaling behaviour of the flux noise

$$S_\Phi(\omega, l, T') = e^{z\delta} S_\Phi(\omega e^{z\delta}, l e^{-\delta}, T'(\delta)) \,. \tag{5}$$

At $\delta^* = \ln(\xi/\xi_0)$ all the vortices are integrated out up to a distance of the order of ξ. At this scale only free vortices are present and the DH approximation can be used to calculate the r.h.s. of Eq. (5).

The effect of screening due to the presence of free vortices above $T_{\rm BKT}$ is analyzed in the DH approximation. The Matsubara Green's function for the vortices in this approximation is

$$G(k, \omega_\mu) = \langle vv \rangle_{k,\omega_\mu} = \frac{1}{4\pi^2 E_J \xi^2 + D(k, \omega_\mu)} \,. \tag{6}$$

The Kosterlitz correlation length ξ diverges exponentially near the transition, $\xi = \xi_0 \exp(b/\sqrt{T' - T'_{\rm BKT}})$, with $T' = T/E_J$ [2, 7, 20]. An analytic continuation ($|\omega_\mu| \to -i\omega$) yields the retarded vortex propagator $G^R(k,\omega) = G(k, |\omega_\mu| \to -i\omega)$ which is related to the spectral function of vortex density-density correlations by the fluctuation-dissipation theorem $\langle vv \rangle_{k,\omega} = {\rm Im}\, G^R(k,\omega)\, 2T/\omega$, for $\omega \ll T$. This correlation function describes the equilibrium vortex fluctuations in the system and gives rise to the flux noise.

Experiments on the magnetic flux noise detect the fluxes of the vortices from an effective area l^2 [6 – 8]. The relevant quantity is the vortex density fluctuation integrated over the pick-up area $\Phi(t) = \int_l d^2x \; v(\mathbf{x}, t)$. After having performed the analytic continuation and by combining the scaling arguments (see Eq. (5)) with the DH approximation (see Eq. (6)), we obtain the flux-noise spectrum

$$S_\Phi(\omega) = \Phi_0^2 \; e^{z\delta^*} S_{\Phi,{\rm DH}}(\omega e^{z\delta^*}, l e^{-\delta^*}, T'(\delta^*)) =$$

$$= \frac{C \; \Phi_0^2}{8\pi^3} \frac{T'(\delta^*) l^4}{\omega \xi^4} F\left(x = \frac{\omega}{\omega_\xi}, y = \frac{4\pi\xi^2}{l^2}\right) \,, \tag{7}$$

where the subscript DH means at at this scale the flux noise can be evaluated by means of the Debye – Hückel approximation. The constant C takes into account the geometrical details of the experimental setup. We introduced the scaling function F:

$$F\left(x = \frac{\omega}{\omega_\xi}, y = \frac{4\pi\xi^2}{l^2}\right) = -{\rm Im} \int_0^y dz \left[1 + z \frac{z - ix}{z - 2ix}\right]^{-1} \,, \tag{8}$$

where characteristic frequency is $\omega_\xi = \omega_0 (\xi_0/\xi)^2$ (we used a hard cutoff in the k-space to integrate over the pick-up coil area). The dynamic exponent $z = 2$ directly emerges. The frequency scale ω_0 is related to microscopic parameters, $\omega_0 = 2\pi E_J/(\xi_0^2 \alpha)$. The scale of the correlation length, ξ_0, is of the order of the lattice spacing. Close to the transition ($\xi \to \infty$) the scaling function F reduces to

$$F(x, y \to \infty) = {\rm Im} \frac{1 + 3ix}{2Q(x)} \ln\left(\frac{1 - ix + Q(x)}{1 - ix - Q(x)}\right) - \frac{\pi}{4} \tag{9}$$

where $Q(x)^2 = 1 - x^2 + 6ix$. The flux noise in the relevant limits is given by

$$\omega S_\Phi(\omega) \sim \begin{cases} \dfrac{\mathcal{C}\,\Phi_0^2}{8\pi^3}\dfrac{T'(\delta^*)l^4}{\xi^4} & \text{for } \dfrac{\omega}{\omega_\xi} \gg 1 \\ \dfrac{\mathcal{C}\,\Phi_0^2}{8\pi^3}\dfrac{T'(\delta^*)l^4}{\xi^4}\dfrac{\omega}{\omega_\xi} & \text{for } \dfrac{\omega}{\omega_\xi} \ll 1\,. \end{cases} \tag{10}$$

The white noise occurs for frequencies $\omega < \omega_\xi$, and $1/f$-noise for $\omega > \omega_\xi$. The $1/f$-noise stems from the superposition of Lorentzian-shaped contributions at all length scales down to the dimension of the SQUID. For finite ξ the $1/f$-noise crosses over to a $1/f^2$-noise. The $1/f$-noise is intermediate in the frequency range $1 < \omega/\omega_\xi < 4\pi\xi^2/l^2$. Close to the transition this range exceeds several orders of magnitude.

We finally apply our results to the experiment by Shaw et al. [7]. The critical coupling in the experiment is $T'_{\rm BKT} = 0.06$, thus the bare fugacity at criticality is $y_0 = 0.35$. Assuming that the bare fugacity is only weakly temperature dependent, the numerical iteration of the RGE gives to a good approximation $T'(\delta) \propto \exp(4\delta)$, as upon renormalization the limit $T'(\delta) \gg 2/\pi$ is reached, where $y(\delta) \propto \exp(2\delta)$ (this scaling behavior reflects the fact that the vortex-pair density is constant). Thus $T'(\delta^*)/\xi^4$ is temperature independent and we can identify our scaling function F with the experimental scaling behavior. These last considerations should be considered more as additional assumptions whose validity is mostly based in the comparison with the experiments which can be made quantitative. The characteristic frequency was found to be $f_0 = 2.1 \cdot 10^6$ Hz. With the estimate $E_J = 10$ K we can extract $\xi_0^2\alpha = 10^6$, corresponding to ohmic shunts to the ground with a resistance $R = 6\,\mathrm{m}\Omega \times \xi_0^2$. We expect ξ_0 to be of the order of one lattice spacing. Yet another possibility is to determine the resistance R and deduce the value of ξ_0. The effective shunting resistance of one island to infinity can be estimated be of the order of $10\,\mathrm{m}\Omega$. This also yields the scale of the correlation length to be of the order of one lattice spacing.

In summary we calculated the flux noise above the BKT transition and derived its scaling behavior analytically in the case where the correlation length exceeds the size of the pick-up loop. We find a crossover from white to $1/f$-noise. We related the crossover frequency to microscopic parameters of the array. Our results may explain a recent experiments by Shaw et al. [7].

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