Pis'ma v ZhETF, vol.63, iss.11, pp.841 - 843 © 1996 June 10

CORRELATION FUNCTION OF SPECKLE IN REFLECTION FROM PHOTONIC PAINT

A.A.Burkov, A.Yu.Zyuzin

A.F.Ioffe Physicotechnical Institute RAS

194021 St.-Petersburg, Russia

Submitted 26 March 1996

We consider the reflection of light from a disordered amplifying medium and calculate the speckle correlation function near the lasing threshold. We also consider the random amplifier–random generator phase diagram on the plane of amplification length versus radiation mean free path.

PACS: 43.30.-d

There is a growing interest in active photonic paints [1, 2]. A photonic paint is a coherently amplifying medium in which light undergoes random scattering. The scattering is provided by small particles [1] or by boundaries of powder grains [2] and can be characterized by a radiation mean free path l. At a high enough gain α this system reaches the lasing threshold and becomes a random laser [3]. Although the idea of applying random scattering as a feedback mechanism [3] is well accepted, the relation between the threshold value of α and l is still a controversial matter [4].

When the system is close to the lasing threshold, a very narrow peak appears in reflection in the backscattering direction [5]. Experiment [2] confirms this finding. To study this peak one has to average out the speckle arising random scattering processes. Here we study the properties of the speckle pattern arising in reflection from a disordered amplifyimg medium and discuss the steady-state amplifier–generator phase diagram on the α–l plane.

Let us consider light scattering by a disordered slab with thickness d and mean free path l. We assume that the longitudinal dimensions of the slab are larger than d and l. The reflection coefficient is related to the sum of the amplitudes of waves reflected along different paths i in the slab as follows:

$$R(\mathbf{k}_0;\mathbf{k}_1) = \left|\sum_i A_i(\mathbf{k}_0;\mathbf{k}_1)\exp\{\alpha L_i\}\right|^2 . \qquad (1)$$

Here $\mathbf{k}_0$ and $\mathbf{k}_1$ are the wave vectors of the incident and scattered wave, respectively, L_i is the length of path i, and $A_i(\mathbf{k}_0;\mathbf{k}_1)$ is the amplitude of scattering along path i in a system without amplification.

The phase difference between the amplitudes $A_i(\mathbf{k}_0;\mathbf{k}_1)$ in a disordered medium is a random quantity, and therefore $R(\mathbf{k}_0;\mathbf{k}_1)$ is a random function of $\mathbf{k}_0$ and $\mathbf{k}_1$. We consider fluctuations of the reflection which are described by the correlation function $\langle R^{(1)}R^{(2)}\rangle$, where $R = R - \langle R\rangle$ and the angle brackets denote averaging over disorder. $R^{(1)}$ and $R^{(2)}$ are the coefficients of reflection of plane waves with wave vectors $\mathbf{k}_0$ and $\mathbf{k}'_0$ into states $\mathbf{k}_1$ and $\mathbf{k}'_1$, respectively.

In considering the fluctuations of the reflection we restrict ourselves to the approximation of uncorrelated diffusion modes. In this case the correlation function is equal to

$$\langle \delta R^{(1)} \delta R^{(2)} \rangle \approx$$

$$\approx \left\langle \sum_i A_i^{(1)} \left(A_i^{(2)*} + A_{-i}^{(2)*} \right) \exp\{\alpha L_i\} \right\rangle \left\langle \sum_i A_i^{(2)} \left(A_i^{(1)*} + A_{-i}^{(1)*} \right) \exp\{\alpha L_i\} \right\rangle . \quad (2)$$

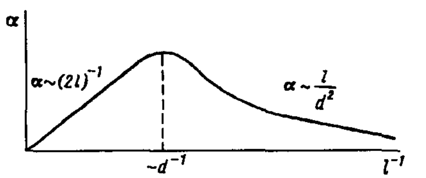
The second term in parentheses is the contribution from a wave which travels along path i but in the opposite direction. This contribution is relevant in a small cone near the backscattering direction [5].

The evaluation of (2) can be done following Refs.[5, 6]. Close to the lasing threshold we obtain the expression

$$\frac{\langle \delta R^{(1)} \delta R^{(2)} \rangle}{\langle R \rangle} = \left| \frac{S\left(\mathbf{k}_0 - \mathbf{k}_0' - \mathbf{k}_1 + \mathbf{k}_1'\right)}{S(0)} \right|^2 \times$$

$$\times \left| \left(1 + \left(\frac{\mathbf{k}_0 - \mathbf{k}_0' + \mathbf{k}_1 - \mathbf{k}_1'}{2k} \right)^2 \right)^{-1} + \left(1 + \left(\frac{\mathbf{k}_0 + \mathbf{k}_0' + \mathbf{k}_1 + \mathbf{k}_1'}{2k} \right)^2 \right)^{-1} \right|^2 . \quad (3)$$

Here $S(\mathbf{k}) = \int_S d\mathbf{r} \exp\{i\mathbf{k}\mathbf{r}\}$ is the form factor of reflecting surface S, $\langle R \rangle > 1$ is the average coefficient of reflection into large angles, $k = p(l,d)\sqrt{\frac{\delta\alpha}{\alpha_0(l,d)}}$, and $\delta\alpha \equiv \alpha_0 \alpha \ll \alpha_0$, where $\alpha_0(l,d)$ is the critical value of the gain. The values of $\alpha_0(l,d)$, $p(l,d)$ and $\langle R \rangle$ depend on the system under consideration and will be determined below.



Schematic drawing of the random amplifier–random generator phase diagram

Expression (3) is valid when k^{-1} is smaller than the dimensions of the surface. In the case when k^{-1} is larger than the dimensions of the surface of the slab, the fluctuations are given by the expression:

$$\frac{\langle \delta R^{(1)} \delta R^{(2)} \rangle}{\langle R \rangle} =$$

$$= \left| \frac{S\left(\mathbf{k}_0 - \mathbf{k}_0' - \mathbf{k}_1 + \mathbf{k}_1'\right) \left\{ S\left(\mathbf{k}_0 - \mathbf{k}_0' + \mathbf{k}_1 - \mathbf{k}_1'\right) + S\left(\mathbf{k}_0 + \mathbf{k}_0' + \mathbf{k}_1 + \mathbf{k}_1'\right) \right\}}{S^2(0)} \right|^2 \quad (4)$$

Let us consider the amplifier–generator phase diagram on the α–l plane and determine the parameters in expressions (3) and (4). The phase diagram is shown schematically in figure. The curve $\alpha = \alpha_0(l,d)$ separates the regions of generation and amplification. In the region below the curve the slab is a random amplifier and on the upper side it is a generator. One can interpret this curve as follows. The amplitude $A_i(\mathbf{k}_0; \mathbf{k}_1) \exp\{\alpha L_i\}$ at the lasing threshold must be finite at $L_i \to \infty$. This means that the factor $\exp\{\alpha L_i\}$ has to compensate for the escape of the wave from the slab. For $d > l$ the motion of the wave in a medium without amplification is diffusive and the wave escapes from the slab after traveling a path length of order $\frac{d^2}{l}$, and therefore the amplitudes $A_i(\mathbf{k}_0;\mathbf{k}_1)$ are exponentially small at $L_i > \frac{d^2}{l}$, and the critical value of the gain is $\alpha_0(l,d) = \frac{1}{6l}\left(\frac{\pi l}{d}\right)^2$ [3]. In this regime the parameters in (3) and (4) are $p(l,d) = \frac{\pi}{d}$ and $\langle R \rangle = \frac{\pi l^2}{4d^3 \delta\alpha}$ [5]. When the thickness of the slab is smaller than the mean free path of the radiation $d < l$, impurities mainly scatter the radiation out of the system, so that the amplitudes $A_i(\mathbf{k}_0;\mathbf{k}_1)$ are exponentially small for $L_i > l$. The critical value of the gain in this case is of the order of the inverse mean free path: $\alpha_0(l,d) \approx \frac{1}{2l}\left(1 - \frac{l}{d}\exp\left\{-\frac{2l}{d}\right\}\right)$ [5]. Generation is easier to achieve when the mean free path is larger. The other parameters in (3) and (4) are $p(l,d) \approx 2\sqrt{\frac{2\alpha_0}{d}\exp\left\{-\frac{2l}{d}\right\}}$ and $\langle R \rangle \approx \frac{\pi}{d\delta\alpha}\exp\left\{-\frac{2l}{d}\right\}$ [5].

At the lasing threshold the wave spends an infinite time inside the slab, and therefore a discrete structure of the modes appears. In this regime the approximation of uncorrelated diffusive modes fails. An experimental manifestation of this regime is a resonance structure arising in the dependence of the reflection coefficient on the frequency of the incident light. The distribution of eigenmodes resembles that of chaotic Hamiltonian systems [7]. Let us note that here and above we assume that the spectral range of amplification is large enough so that the system supports many lasing modes.

An interesting manifestation of a discrete structure of levels in a disordered system is the dynamic echo, which arises due to level repulsion [8]. Although it was proposed for electrons in a quantum dot, dynamic echo occurs also in an amplifying disordered medium as follows. Consider radiation from a slab after the injection of a short pulse at some point on the surface. Diffusing throughout the slab, the radiation will gradually become uniform. But eventually the point of injection again becomes brighter than the background at a time which equals the inverse of the intermode separation. Since this time is proportional to the volume of the slab, it is more feasible from an experimental standpoint to study the echo in a small slab in order to avoid dephasing processes. Because the dynamic echo appears only on average, to study it one needs to scan over the frequency of the injected light or, for a given frequency, to scan over the point of incidence.

We thank V.Prigodin for helpful discussions.

This work is supported by Russian Fund for Fundamental Research, Grant 26-02-16902-a.

---

1. N.M. Lawandy, R.M. Balachandran, A.S.L.Gomes, and E.Sauvain, Nature **386**, 436 (1994).
2. D.S.Wiersma, M.P. van Albada, and A.Lagendijk, Phys. Rev. Lett. **75**, 1739 (1995).
3. V.S.Letokhov, Zh. Eksp. Teor. Fiz. **53**, 1442 (1967).
4. A.Z.Genack and J.M.Drake, Nature **386**, 400 (1994).
5. A.Yu.Zyuzin, Europhys. Lett. **26**, 517 (1994); Pis'ma Zh. Eksp. Teor. Fiz. **61**, 961 (1995).
6. D.Eliyahu, R.Berkovitz, and M.Kaveh, Phys. Rev. B **43**, 13501, (1991).
7. A.Yu.Zyuzin, Phys. Rev. E **51**, 5274 (1995).
8. V.N.Prigodin, B.L.Altshuler, K.B.Efetov, and S.Iida, Phys. Rev. Lett. **72**, 546 (1994).