

OBSERVATION OF PARAMETRIC AMPLIFICATION OF PROPAGATING DIPOLE-EXCHANGE SPIN WAVES IN YTTRIUM IRON GARNET FILMS

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Amplification of a weak-signal spin wave by a strong pump spin wave propagating in an yttrium iron garnet film was observed for the first time. Theoretical explanation of the amplification phenomenon is suggested on the basis of four-wave parametric processes.

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The well-known mechanisms which could be used to amplify spin waves (SW) in magnetic crystals are perpendicular and parallel microwave pumping [1]. In both cases pumping frequency should be two times bigger than the frequency of the amplified wave. It is very tempting to incorporate some amplification mechanism in the variety of nonlinear phenomena in ferromagnetic films, especially in spin-wave soliton phenomena which are actively studied during the last decade or so (see e.g. [2-4] and references therein). The aim of this paper is to report the first experimental observation of the amplification of a weak (signal) spin wave by a strong (pump) spin wave propagating in a ferromagnetic film at adjacent frequencies and to suggest a theory to describe the experimental results.

When two or more SW propagate in a ferromagnetic film they can interact with each other through the spin-system nonlinearity. The main idea of the experiment was to use a four-wave parametric nonlinear process causing modulation instability of a pump SW of frequency f_p to amplify the signal SW having its carrier frequency f_s shifted from f_p by roughly the autooscillation frequency Ω_m of modulation instability. The measurements utilized a delay-line consisting of two short-circuited microstrip transducers (width 50 μm, distance between transducers l =4 mm). For the experiments we used perpendicularly magnetized yttrium iron garnet (YIG) film strips with widths of 1 and 2 mm. The YIG strips were cut from a single-crystal (111)-oriented 5.2 μm thick film. The YIG film was chosen to have mixed exchange boundary conditions (surface spin pinning conditions) to provide a possibility for varying the SW dispersion by tuning the carrier frequency [2,3].

A comparison of the experimentally taken in the low-power $P_s = -34.5$ dBm cw regime transmission loss characteristic (Fig.1a,c) and calculations from dipole-exchange theory [5] (Fig.1b,d) indicates that there were dipole "gaps" and consequently regions of high dispersion in the dipole-exchange spin-wave spectrum of the experimental sample.

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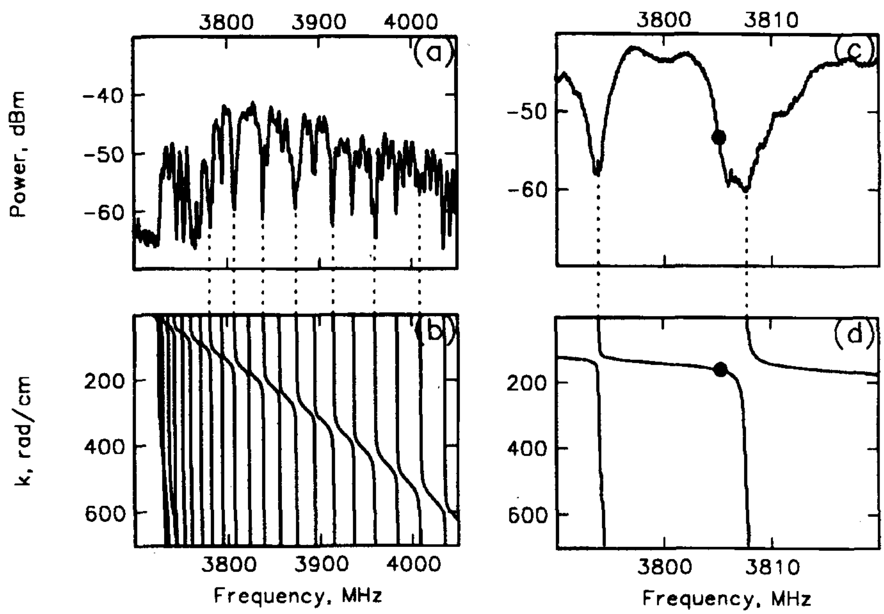

Fig.1. Low power transmission loss vs. frequency characteristic and calculated spin-wave spectrum: (*a*) and (*b*) represent full-scale frequency data, (c) and (d) are their enlarged fragments done in the vicinity of the operating frequency point f_p =3805 MHz which is marked by filled circle. The parameters used for the calculation are: film thickness L =5.2μm, external magnetic field H =3060 Oe, saturation magnetization $4\pi M_0$ =1750 G, exchange constant α =3.1$\cdot 10^{-12}$ cm^2, surface spin pinning parameters $d_1 L$ =500, $d_2 L$ =17

As in the previous work [2,3], to obtain modulation instability of the pump SW its carrier frequency was situated in the regions of high dispersion near the dipole "gaps" in the spin-wave spectrum. In these regions of high dispersion the criterion demanding opposite signs of the SW dispersion D and nonlinearity T is satisfied (in the case under consideration $D < 0$, $T > 0$). It was observed that an increase of the input power P_p first induced a modulation instability in the narrow frequency intervals situated on the low-frequency slopes of the dipole "gaps" where $D < 0$. The autooscillation frequencies of the quasi-harmonic instability appearing near the threshold were measured to be about 3-6 MHz. In addition, with gradual increase of P_p above the threshold a very complex multistable behavior, a variety of oscillations and sequences of bifurcations were observed. To characterize the nonlinear regime in detail a special study is necessary.

The basic experiment was performed as follows. In order to simplify the search for the necessary combination of parameters (signal and pumping frequencies, pump wave amplitude, etc.), first the pump wave was launched and its modulation instability was registered. During these measurements the threshold power P_{th} and the frequency f_m of the quasi-harmonic instability were determined. Then for the values of the input power P_p chosen to be below and above P_{th} an amplification of a signal spin wave having a carrier frequency f_s situated near f_m was studied.

Fig.2 shows a series of spectra taken at the output transducer for the perpendicularly magnetized (H =3060 Oe) YIG strip of 2 mm width. The upper spectrogram (5) corresponds to the regime of modulation instability when only one pump monochromatic wave was launched in the YIG strip at the frequency f_p = 3805 MHz. The threshold power was measured to be P_{th} =5.5 dBm and the autooscilation frequency $\Omega_m = f_p - f_m$ =3.2 MHz. Spectrograms (1)-(4) of Fig.2 illustrate SW amplification. The carrier frequency of the signal wave was taken smaller than f_m and equaled to 3802.76 MHz. Spectrogram (1) shows the spectral line of the week-signal wave (input power P_s =-34.5 dBm) in the absence of the pump wave. Spectrograms (2), (3), and (4) were recorded in the amplification regime when both pump and signal spin waves copropagated in the film. These spectrograms were taken for the input power level P_p =1.5 dBm, P_p =5.1 dBm, and P_p =6.2 dBm, respectively. To provide a possibility to judge about an amplification rate, a part of transmission loss characteristic (recorded in the linear regime for the input-signal power level P_s =-34.5 dBm) is also given on these spectrograms. As evident from the spectrograms (1)-(4), parametric amplification of the weak spin wave by the strong spin wave takes place. The amplification exists in a range of pumping powers below (spectrograms (2), (3)) and above (spectrogram (4)) the threshold of modulation instability. Maximum amplification gain was more than 20 dB for P_s=-34.5 dBm and P_p =5.1 dBm.

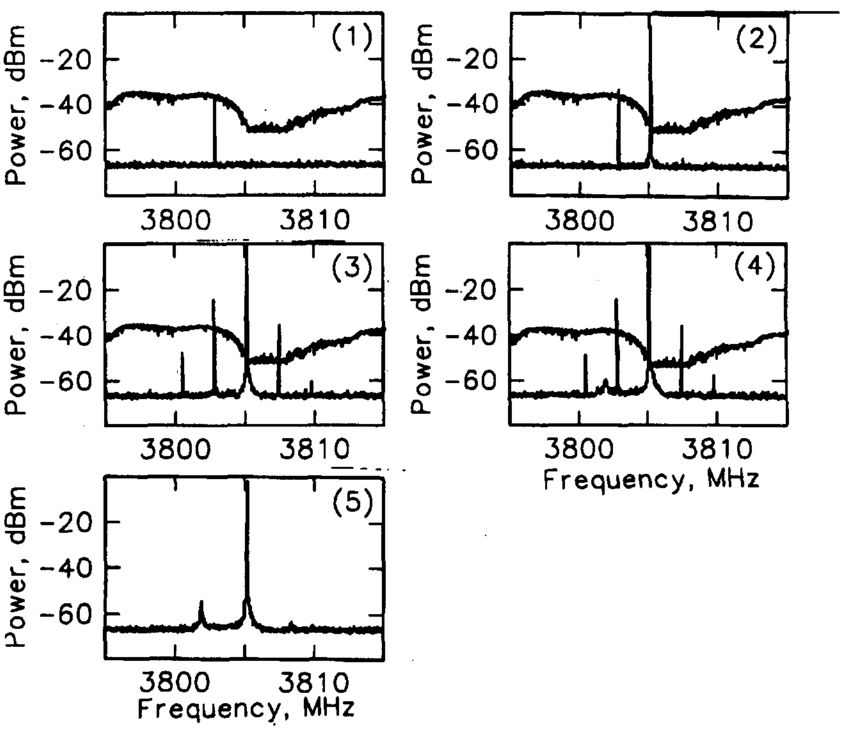

Fig.2. Spectra of monochromatic input signals at the YIG film output obtained for different conditions: *(1)* P_p =0, P_s =-34.5 dBm; *(2)* P_p =1.5 dBm, P_s =-34.5 dBm; *(3)* P_p =5.1dBm, P_s =-34.5 dBm; *(4)* P_p =6.5 dBm, P_s =-34.5 dBm; *(5)* P_p =6.2 dBm, P_s =0

Theoretical explanation of the observed amplification phenomenon is suggested on the basis of the four-wave parametric process

$$2\omega_p(k_0) = \omega_s(k_0 - \kappa) + \omega_i(k_0 + \kappa) \quad (1)$$

where we assume for definiteness $\omega_s < \omega_i$ and that the wave vector of perturbation κ is small compared with carrier wave vectors, i.e. $\kappa \ll k_0$. Applying the Hamiltonian formalism a system of coupled equations for the amplitudes of the pump wave, the amplified wave, and the idle wave was derived. We started with the energy of the film taking into account Zeeman, dipole-dipole and exchange interactions as well as electrodynamic and exchange boundary conditions. We used circularly polarized canonical variables in which the energy becomes a Hamiltonian function [6] , and proceeded similarly to [7] but utilizing a rigorous procedure [8] for the diagonalization of the quadratic part of the Hamiltonian function , and a similar procedure for the exclusion of non-resonant three-wave terms. In the result we obtain a system of equations for the Fourier-amplitudes of the amplified C_{1k} and idle C_{2k} waves coupled by the pump wave. Then applying the inverse Fourrier transform and taking into account the conservation law (1) we passed to the coordinate presentation. For this purpose we expanded SW dispersion law $\omega(k)$ into the Taylor series. Retaining first three lower-order terms we wrote

$$\omega(k) = \omega(k_0) + \kappa V_g(k_0) + \kappa^2 D(k_0)/2 \quad . \tag{2}$$

The above described calculations led us to the below given equations (3)-(5). The equation for the slow amplitude $c_0(z) = C_0(z)\exp(-ik_0 z)$ of pump wave has a form of the nonlinear Shrödinger equation with a damping term

$$i[\frac{\partial}{\partial t} + V_g\frac{\partial}{\partial z} + \gamma]c_0(z) + \frac{D}{2}\frac{\partial^2 c_0(z)}{\partial z^2} - T|c_0(z)|^2 c_0(z) = 0 \quad . \tag{3}$$

Here $T = T(k_0, k_0; k_0, k_0)$ is the four-wave interaction coefficient, γ is the spin-wave dissipation parameter.

Solutions of equation (3) neglecting relaxation are well known. The solution with the minimum energy $E = E_{min}$ corresponds to a wave with the nonlinear frequency shift. For $E > E_{min}$ periodic oscillations (modulation instability) may appear (see e.g. [6]).

Under the threshold of modulation instability of the pump wave the system of equations for the slow amplitudes of the amplified wave $c_1(z) = C_1(z)\exp(-ik_0 z)$ and the idle wave $c_2(z) = C_2(z)\exp(-ik_0 z)$ has the form

$$\begin{aligned} i[V_g\frac{\partial}{\partial z} + \gamma]c_1(z) + \frac{D}{2}\frac{\partial^2}{\partial z^2}c_1(z) - (2T|c_0(z)|^2 + \\ +\Delta\omega)c_1(z) - Tc_0^2(z)c_2^*(z) = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} -i[V_g\frac{\partial}{\partial z} + \gamma]c_2^*(z) + \frac{D}{2}\frac{\partial^2}{\partial z^2}c_2^*(z) - (2T|c_0(z)|^2 - \\ -\Delta\omega)c_2^*(z) - Tc_0^{*2}(z)c_1(z) = 0 \end{aligned} \tag{5}$$

where $\Delta\omega = \omega_p - \omega_s$.

In the case $\gamma = 0$ we have $c_0(z) = const(z) = c_0$, and an analytical solution of the system (3)-(4) exists. It may be written in the form $c_1(z) = c_1 exp(\nu z)$, $c_2^*(z) = c_2^* exp(\nu z)$. Substitution of this solution into equations (4), (5) gives the dispersion relation:

$$\nu^4 + (\frac{4V_g^2}{D^2} - \frac{4T|c_0|^2}{D})\nu^2 + i\frac{8V_g}{D^2}\Delta\omega\nu - \frac{4}{D^2}\Delta\omega^2 = 0 \quad . \tag{6}$$

Note that here V_g and D are calculated for $\omega(k_0) = \omega_p - Tc_0^2$.

Only two roots of (5) lie in the range of κ-values where the expansion for SW dispersion (2) is valid. Moreover, only these two roots satisfy the condition $Re(\nu) < 0$ in the limiting case $|c_0|^2 \rightarrow 0$ when a small SW dissipation $\gamma > 0$ is introduced in (6). A positive value of the real part of one of the significant roots of (6) corresponds to the amplification regime whereas both negative values correspond to the signal decay regime. We stress that in the frequency range where expansion (2) is valid the positive values of $Re(\nu)$ are possible only in the case when $DT < 0$.

A numerical procedure was elaborated to solve the system (3)-(4) in the case $\gamma \neq 0$. In particular, the amplification gain coefficient $G = 10\log P_{sw}(z=l)/P_{sw}(z=0)$ was calculated (here $P_{sw}(z=l)$ and $P_{sw}(z=0)$ are the powers of the signal spin wave at the output and the input microstrip transducers respectively).

Numerical calculations show that the gain depends strongly and non-monotonously on a combination of several factors such as the SW propagation pass l, the initial amplitude of pumping wave $c_0(z=0)$, the value of microwave magnetic damping, and the frequencies f_p and f_s. Depending on f_p and initial amplitude $c_0(z=0)$ the maximum of $G(f_s)$ may appear either in the vicinity of the frequency $f_p - \Omega_m$, or at the frequency f_p . Depending on $c_0(z=0)$ and film parameters, G as a function of distance l may have a maximum or decrease monotonously. The results of detailed numerical analysis will be published elsewhere. Here we will only compare results of our calculations with the experimental data obtained for the amplification gain.

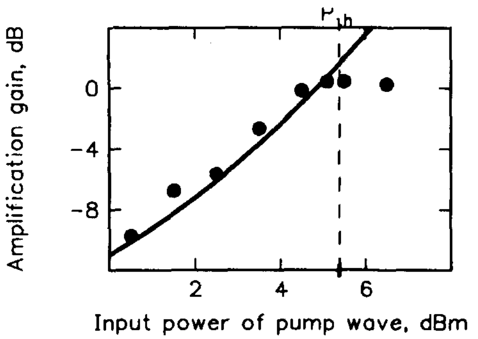

Fig.3. Amplification gain as a function of input power of the pump wave for perpendicularly magnetized YIG film of thickness 5.2 μm (filled circles - experiment, solid line - theory). Experimental parameters are: external magnetic field 3060 Oe, pump frequency f_p =3805 MHz, signal frequency f_s =3802.7 MHz, signal wave input power P_s = -34.5 dBm. The calculations are made for the experimental set of parameters using the assumption that surface spins are totally pinned and exchange constant α =3.1·10^{-12}cm^2

A result of numerical calculations for the set of parameters corresponding to the above described experimental situation is shown in Fig.3. This figure shows the dependence of the amplification gain on the input power of the pump spin wave P_p. (We note that the group velocity $V_g(f_s)$ of spin waves and the dispersion coefficient $D(f_s)$ were calculated starting with the dipole-exchange spin-wave spectrum of a film with totally pinned surface spins. Within the pumping power range 0.5...6.5 dBm which was used to calculate the theoretical dependence $G(P_p)$ in Fig.3 the group velocity and the dispersion coefficient were changed by a nonlinear spectrum shift in the limits $1.0 \cdot 10^6 ... 2.0 \cdot 10^6$ cm/s and $-6.0 \cdot 10^4 ... -4.3 \cdot 10^4$ cm^2·rad/s respectively.) Filled circles in Fig. 3 are the experimental points. As seen from the figure, a discrepancy between the theory and the experiment appears for the comparatively large values of P_p. The reason for this could be an onset of modulation instability of the pump wave which was not taken into account in the calculations.

In conclusion, parametric amplification of a weak SW by a strong SW propagating in YIG film at adjacent microwave frequencies lying in the highly dispersive zone of dipole-exchange spin-wave spectrum was experimentally observed and theoretically explained. The theory predicts that a similar amplification effect could be observed for low dispersive (purely dipole) spin waves.

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