Pis'ma v ZhETF, vol.65, iss.1, pp.9 - 14

THE Q^2 DEPENDENCE OF THE MEASURED ASYMMETRY A_1: THE TEST OF THE BJORKEN SUM RULE

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Submitted 11 September 1996
Resubmitted 4 December 1996

We analyse the proton and deutron data on spin dependent asymmetry $A_1(x, Q^2)$ supposing the DIS structure functions $g_1(x, Q^2)$ and $F_3(x, Q^2)$ have the similar Q^2-dependence. As a result, we have obtained that $\Gamma_1^p - \Gamma_1^n = 0.190 \pm 0.038$ at $Q^2 = 10$ GeV^2 and $\Gamma_1^p - \Gamma_1^n = 0.165 \pm 0.026$ at $Q^2 = 3$ GeV^2, what is in the best agreement with the Bjorken sum rule predictions.

PACS: 11.55.Hx, 13.60.Hb, 13.88.+e

An experimental study of the nucleon spin structure is realized by the measuring of asymmetry $A_1(x, Q^2) = g_1(x, Q^2)/F_1(x, Q^2)$. The best known theoretical predictions on spin dependent structure function $g_1(x, Q^2)$ of the nucleon were made by Bjorken [1] and Ellis and Jaffe [2] for the so called *first moment value* $\Gamma_1 = \int_0^1 g_1(x)dx$.

The calculation of the Γ_1 value requires the knowledge of structure function g_1 at the same Q^2 in the whole x range. Experimentally the asymmetry A_1 is measuring at different values of Q^2 for different x bins. An accuracy of the past and modern experiments [3, 4] allows to analyze data in the assumption [5] that asymmetry $A_1(x, Q^2)$ is Q^2 independent (i.e. the structure functions g_1 and F_1 have the same Q^2 dependence). However, this assumption is not theoretically warranted (see discussions in [6–8]); the different Q^2 dependence of the structure functions $g_1(x, Q^2)$ and $F_1(x, Q^2)$ is expected due to the difference in polarized and unpolarized spliting functions (except for the leading order quark-quark one). Thus, in view of forthcoming more precise data it is important to add the Q^2 dependence of the asymmetry.

This article is based on our observation that the Q^2 dependence of spin dependent and spin average structure functions g_1 and F_3 is very similar in a wide x range: $10^{-2} < x < 1$. At the small x region ($x < 10^{-2}$ it could be not true (see [6, 9]), but most of the existed data were measured out off that range.

Lets consider the nonsinglet (NS) Q^2 evolution of structure functions F_1, g_1 and F_3. The DGLAP equation for the NS part of these functions can be presented as[3]:

$$\frac{dg_1^{NS}(x, Q^2)}{dlnQ^2} = -\frac{1}{2}\bar{\gamma}_{NS}(x, \alpha) \times g_1^{NS}(x, Q^2),$$

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[3] We use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

$$\frac{dF_1^{NS}(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\gamma^{+}_{NS}(x,\alpha)\times F_1^{NS}(x,Q^2), \qquad (1)$$
$$\frac{dF_3(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\gamma^{-}_{NS}(x,\alpha)\times F_3(x,Q^2),$$

where symbol $\times$ means the Mellin convolution. The spliting functions $\gamma^{\pm}_{NS}$ are the reverse Mellin transforms of the anomalous dimensions $\gamma^{\pm}_{NS}(n,\alpha) = \alpha\gamma^{(0)}(n)_{NS} + \alpha^2\gamma^{\pm(1)}_{NS}(n) + O(\alpha^3)$ and the Wilson coefficients[4] $\alpha b^{\pm}(n) + O(\alpha^2)$:

$$\gamma^{\pm}_{NS}(x,\alpha) = \alpha\gamma^{(0)}_{NS}(x) + \alpha^2\left(\gamma^{\pm(1)}_{NS}(x) + 2\beta_0 b^{\pm}(x)\right) + O(\alpha^3), \qquad (2)$$

where $\beta(\alpha) = -\alpha^2\beta_0 - \alpha^3\beta_1 + O(\alpha^4)$ is QCD β-function.

The above mentioned Mellin transforms mean that

$$f(n,Q^2) = \int_0^1 dx x^{n-1} f(x,Q^2), \qquad (3)$$

where $f = \{\gamma^{(0)}_{NS}, \gamma^{\pm(1)}_{NS}, b^{\pm}_{NS}, \gamma^{(k)}_{ij}, \gamma^{*(k)}_{ij}, b_i$ and $b^*_i\}$ with $k = 1,2$ and $\{i,j\} = \{S,G\}$.

Eqs. (1) show the Q^2 dependence of NS parts of g_1 and F_3 is the same (at least in first two orders of the perturbative QCD [10]) and differs from F_1 already in the first subleading order $\left(\gamma^{+(1)}_{NS} \neq \gamma^{-(1)}_{NS}\right.$ [11] and $\left. b^{+}_{NS} - b^{-}_{NS} = (8/3)x(1-x)\right)$.

For the singlet parts of g_1 and F_1 evolution equations are :

$$\frac{dg_1^S(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\left[\gamma^*_{SS}(x,\alpha)\times g_1^S(x,Q^2) + \gamma^*_{SG}(x,\alpha)\times \Delta G(x,Q^2)\right],$$
$$\frac{dF_1^S(x,Q^2)}{dlnQ^2} = -\frac{1}{2}\left[\gamma_{SS}(x,\alpha)\times F_1^S(x,Q^2) + \gamma_{SG}(x,\alpha)\times G(x,Q^2)\right], \qquad (4)$$

where

$$\gamma_{SS}(x,\alpha) = \alpha\gamma^{(0)}_{SS}(x) + \alpha^2\left(\gamma^{(1)}_{SS}(x) + b_G(x)\times\gamma^{(0)}_{GS}(x) + 2\beta_0 b_S(x)\right) + O(\alpha^3),$$
$$\gamma_{SG}(x,\alpha) = \frac{e}{f}\left[\alpha\gamma^{(0)}_{SG}(x) + \alpha^2\left(\gamma^{(1)}_{SG}(x) + b_G(x)\times\left(\gamma^{(0)}_{GG}(x) - \gamma^{(0)}_{SS}(x)\right) + 2\beta_0 b_G(x) + \right.\right.$$
$$\left.\left. + \; b_S(x)\times\gamma^{(0)}_{SG}(x)\right)\right] + O(\alpha^3).$$

where $e = \sum_i^f e_i^2$ is the sum of charge squares of f active quarks. The equations for polarized anomalous dimensions $\gamma^*_{SS}(x,\alpha)$ and $\gamma^*_{SG}(x,\alpha)$ are similar. They can be obtained by replacing $\gamma^{(0)}_{SG}(x) \rightarrow \gamma^{*(0)}_{SG}(x)$, $\gamma^{(1)}_{Si}(x) \rightarrow \gamma^{*(1)}_{Si}(x)$ and $b_i(x) \rightarrow b^*_i(x)$ $(i = \{S,G\})$.

Note here the gluon term is not negligible for F_1 at $x < 0.3$ but for g_1 we can neglect them for $x > 0.01$ [7, 8]. The value $b^*_s(x)$ $(b_s(x))$ coincides

[4]Because we consider here the structure functions themselves but not the parton distributions. Note that $b^{+}_{NS}(n)$ and $b^{-}_{NS}(n)$ have more standard definition as $b_{1,NS}(n) = b_{2,NS}(n) - b_{L,NS}(n)$ and $b_{3,NS}(n)$.

with $b^-(x)$ $(b^+(x))$. The difference between $\gamma_{NS}^{-(1)}$ and $\gamma_{SS}^{*(1)} + b_G^*(x) \times \gamma_{GS}^{(0)}(x)$ is negligible because it does not contain a power singularity at $x \to 0$ (i.e. a singularity at $n \to 1$ in momentum space). Moreover, it decreases as $O(1-x)$ at $x \to 1$ [12]. Contrary to this, the difference between $\gamma_{SS}^{(1)} + b_G(x) \times \gamma_{GS}^{(0)}(x)$ and $\gamma_{SS}^{*(1)} + b_G^*(x) \times \gamma_{GS}^{(0)}(x)$ contains the power singularity at $x \to 0$ (see for example [10]).

The analysis discussed above allows us to conclude the function A_1^* :

$$A_1^*(x) = \frac{g_1(x, Q^2)}{F_3(x, Q^2)} \tag{5}$$

should be practically Q^2 independent at $x > 0.01$.

The r.h.s. of Eqs.(1) and (1) contain integrals of structure functions and, hence, the approximate validity of (5) should be observed only for the similar x-dependence of $g_1(x, Q^2)$ and $F_3(x, Q^2)$ at fixed Q^2. But it is the case (see [13] at $Q^2 = 3GeV^2$, for example).

The asymmetry A_1 at $Q^2 =< Q^2 >$ can be defined than as:

$$A_1(x_i, < Q^2 >) = \frac{F_3(x_i, < Q^2 >)}{F_3(x_i, Q_i^2)} \cdot \frac{F_1(x_i, Q_i^2)}{F_1(x_i, < Q^2 >)} \cdot A_1(x_i, Q_i^2), \tag{6}$$

where x_i (Q_i^2) means an experimentally measured value of x (Q^2).

We use SMC and E143 proton and deuteron data on asymmetry $A_1(x, Q^2)$ [3, 4]. To get $F_1(x, Q^2)$ we take NMC parametrization of $F_2(x, Q^2)$ [14] and SLAC parametrization of $R(x, Q^2)$ [15] $(F_1 \equiv F_2/2x[1+R])$. To get the values of $F_3(x, Q^2)$ we parametrize the CCFR data [16] as a function of x and Q^2 (see the parametrization in Appendix).

First, using Eq.(5), we recalculate the asymmetry measured by SMC [3] and E143 [4] on the proton and deuteron targets at $Q^2 = 10$ GeV^2 (SMC) and 3 GeV^2 (E143), which are average Q^2 of these experiments respectively. Obtained values of $\int g_1(x)dx$ through the measured x ranges are shown in the Table 1.

Table 1

The first moment values of g_1 of the proton and deuteron

$x_{min} - - x_{max}$	$< Q^2 >$	target type	$\int_{x_{min}}^{x_{max}} g_1 dx$	Γ_1	experiment
.003 - 0.7	10 GeV^2	proton	0.130	0.134 $\pm$ 0.011	SMC
.003 - 0.7	10 GeV^2	deuteron	0.038	0.036 $\pm$ 0.009	SMC
.029 - 0.8	3 GeV^2	proton	0.123	0.130 $\pm$ 0.004	E143
.029 - 0.8	3 GeV^2	deuteron	0.043	0.044 $\pm$ 0.003	E143

To get the values of the first moments $\Gamma_1^{p(d)}$ we estimate unmeasured regions of SMC and E143 using their original machinary. Our estimations coincide with original ones except to the results in small x region unmeasured by SMC. We obtain the following results for central values of $\Delta\Gamma^{p,d} = \int_0^{0.003} g_1(x)dx$ at $Q^2 = 10$GeV^2: $\Delta\Gamma^p = 0.003$ and $\Delta\Gamma^d = 0.0022$, which are smaller then the corresponding SMC estimations: $\Delta\Gamma^p = 0.004$ and $\Delta\Gamma^d = 0.0028$. The errors coincide with ones cited in [3]. The E143 estimations for $\int_0^{0.029} g_1(x)dx$ are not changed because Q^2-evolution of the asymmetry is negligible at $x \sim 0.03$. Results on the Γ_1 values are shown also in the Table 1.

We would like to note that the E143 and SMC machinary may lead to underestimation of $g_1^{p,d}(x,Q^2)$ at small x and, hence, to underestimation of $\Delta\Gamma^{p,d}(Q^2)$ (see the careful analysis in first paper in ref. [8]). Unfortunately, our procedure is not at work at $x \leq 0.01$ and we cannot check the SMC and E143 estimations of unmeasured regions here. To clear up this situation it is necessary to add a careful small x analysis to this consideration that is a subject of our future large article [17].

As the last step we calculate the difference which is predicted by the Bjorken sum rule $\Gamma_1^p - \Gamma_1^n$:

$$\Gamma_1^p - \Gamma_1^n = 2\Gamma_1^p - 2\Gamma_1^d/(1 - 1.5 \cdot \omega_D),$$

where $\omega_D = 0.05$ [3, 4] is the probability of the deutron to be in a D-state.

At $Q^2 = 10$ GeV^2 we get the following results:

$$\Gamma_1^p - \Gamma_1^n = 0.190 \pm 0.038 \qquad (7)$$

to be compared with the SMC published value

$$\Gamma_1^p - \Gamma_1^n = 0.199 \pm 0.038 \qquad \text{(SMC [3])}$$

and the theoretical prediction

$$\Gamma_1^p - \Gamma_1^n = 0.187 \pm 0.003 \qquad \text{(Theory)}.$$

At $Q^2 = 3$ GeV^2 we get for E143 data:

$$\Gamma_1^p - \Gamma_1^n = 0.165 \pm 0.026 \qquad (8)$$

to be compared with

$$\Gamma_1^p - \Gamma_1^n = 0.163 \pm 0.026 \qquad \text{(E143 [4])},$$
$$\Gamma_1^p - \Gamma_1^n = 0.171 \pm 0.008 \qquad \text{(Theory)}.$$

Note that only the statistical errors are quoted here. To the considered accuracy they coincide with the errors cited in ([3, 4]). The above cited theoretical predictions for the Bjorken sum rule have been computed in [18] to the third order in the QCD α_s.

As a conclusion, we would like to note:

- The value of $\Gamma_1^p - \Gamma_1^n$ obtained in our analysis is in the best agreement with the Bjorken sum rule prediction.
- The values of Γ_1^p and Γ_1^n themselves obtained here do not change essentially. The improvement for the Bjorken sum rule is the result of the opposite changes of the Γ_1^p and Γ_1^n values, when Eq.(5) is used.
- Our observation that function $A_1^*(x)$ is Q^2 independent at large and intermediate x is supported by good agreement of present analysis with other estimations [19, 7, 8] of the Q^2 dependence of the A_1. A detail analysis will be present later in the separate large article [17].

We are grateful to W.G. Seligman for providing us the available CCFR data of Ref.[16], A.V. Efremov for discussions and to anonimous Referee for critical notices lead to essential improvement of the article. We also grateful to G. Ridolfi for the possibility to present the short version [20] of this letter on polarized structure function section of the Workshop DIS96.

This work is supported partially by the Russian Fund for Fundamental Research, Grant 95-02-04314-a.

Appendix. The parametrization is used for CCFR data [16] :

$$xF_3(x,Q^2) = F_3^a \cdot \left(\frac{\log(Q^2/\Lambda^2)}{\log(Q_0^2/\Lambda^2)}\right)^{F_3^b},$$

where

$$F_3^a = x^{C_1}(1-x)^{C_2}\Big(C_3 + C_4(1-x) + C_5(1-x)^2 +$$

$$+C_6(1-x)^3 + C_7(1-x)^4\Big)\Big[C_8 + C_9 x + C_{10}x^2 + C_{11}x^3\Big],$$

$$F_3^b = C_{12} + C_{13}x + \frac{C_{14}}{x + C_{15}}$$

and $Q_0^2 = 10\ \mathrm{GeV}^2$, $\Lambda = 200$ MeV.

Table 2

The values of the coefficients of CCFR data parametrization

C_1	C_2	C_3	C_4	C_5
0.8064	1.6113	0.70921	-2.2852	1.8927
C_6	C_7	C_8	C_9	C_{10}
6.0810	4.5578	0.7464	-0.3006	3.9181
C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
-0.1166	10.516	-5.7336	-37.114	3.7452

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