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COLLECTIVE EXCITATIONS OF MASSIVE DIRAC PARTICLES IN A HOT AND DENSE MEDIUM

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The one-loop dispersion relation which defines the collective excitations of massive Dirac particles in a hot and dense quark–gluon medium is obtained in the high-temperature limit for the case $m \ll gT$ and is solved explicitly for all $|\mathbf{q}|$ when $\mu = 0$. Four well-separated spectrum branches (quasi-particle and quasi-hole excitations) are found, and their behaviors for small and large $|\mathbf{q}|$ are investigated. All calculations are performed using the temperature Green function technique and fixing the Feynman gauge. The gauge dependence of the spectra found is briefly discussed.

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Introduction. The study of collective excitations in a hot and dense medium is a topical problem in present-day physics, especially for chromodynamics. In a medium all the particles (fermions, as well as bosons) lose their individual properties, and, due to the multi-interaction with the heat bath and one another, collective excitations arise, which (unlike the ordinary vacuum physics at $T, \mu = 0$) have many new peculiarities: a gap of the order of gT at zero momentum, and a split spectrum at finite momentum [1-5]. These collective excitations determine the bulk of the kinetic and thermodynamic properties of the hot and dense medium and are very important for many processes taking place, for example, inside a hot quark–gluon plasma. Moreover, the quark–gluon medium (when μ and/or T are nonzero) generates new collective excitations of fermions: quasi-holes [2,4], which are different from the quasi-particle excitations; their peculiarities (e.g., the minimum of the quasi-hole branches at finite momentum and the "wrong" relation between chirality and helicity) can produce new physical consequences. All these collective modes have nonzero effective masses, which arise dynamically independently of the bare masses and are not small for large parameters T, μ. In particular, for initially massive Dirac particles it is established that there is a set of four effective masses [6,7,8], which, in the general case, are well-separated and are always nonzero in the medium.

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The goal of this paper is to present the one-loop dispersion relation which determines the collective excitations of massive Dirac particles in a hot and dense quark–gluon plasma in the high-temperature limit for the case $m \ll gT$, and to solve it explicitly for all $|\mathbf{q}|$ when $\mu = 0$. We use the standard temperature Green function technique and fix the Feynman gauge for explicit calculations. Only the case of zero damping is considered, and additional problems connected with the damping [5,9] are not discussed. Four well-separated spectrum branches are established, and their behavior for small and large $|\mathbf{q}|$ is investigated. The gauge dependence of the spectra found is briefly discussed. To start, we choose hot and dense QCD, although many results are model independent.

QCD Lagrangian and quark self-energy. The QCD Lagrangian in covariant gauges has the form

$$\begin{aligned} \mathcal{L} = & - \frac{1}{4}G^{a}_{\mu\nu}{}^{2} + N_f\bar{\psi}[\gamma_\mu(\partial_\mu - \frac{1}{2}ig\lambda^a V^a_\mu) + m]\psi - \\ & - \mu N_f\bar{\psi}\gamma_4\psi + \frac{1}{2\alpha}(\partial_\mu V^a_\mu)^2 + \bar{C}^a(\partial_\mu\delta^{ab} + gf^{abc}V^c_\mu)\partial_\mu C^b \end{aligned} \tag{1}$$

where $G^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + gf^{abc}V^b_\mu V^c_\nu$ is the Yang–Mills field strength; V_μ is a non-Abelian gauge field; ψ(and $\bar{\psi}$) are the quark fields in the SU(N) fundamental representation ($\frac{1}{2}\lambda^a$ are its generators and f^{abc} are the SU(N) structure constants), and C^a (and $\bar{C}^a$) are the ghost Fermi fields. In Eq.(1) μ and m are the quark chemical potential and the bare quark mass, respectively, N_f is the number of quark flavors, and α is the gauge-fixing parameter ($\alpha = 1$ for the Feynman gauge). The metric is chosen to be Euclidean, and $\gamma^2_\mu = 1$.

To find a non-perturbative representation for the temperature quark Green function, we start from the exact Schwinger–Dyson equation

$$G^{-1}(q) = G_0^{-1}(q) + \Sigma(q) \tag{2}$$

and calculate the quark self-energy, which in any gauge has the simple, well-known representation [10]

$$\Sigma(q) = \frac{N^2-1}{2N}\frac{g^2}{\beta}\sum_{p_4}^{F}\int\frac{d^3p}{(2\pi)^3}\mathcal{D}_{\mu\nu}(p-q)\gamma_\mu G(p)\Gamma_\nu(p,q|p-q)\,. \tag{3}$$

In what follows we calculate $\Sigma(q)$ only in the one-loop approximation, using the bare Green functions in Eq.(3) and fixing the Feynman gauge (i.e., taking the appropriate $\mathcal{D}$ function). All ultraviolet divergences are renormalized as usual, but the infrared ones (which also arise in the high-temperature expansion when $m \ll gT$) will be eliminated phenomenologically.

At first the summation over the spinor indices is performed in Eq.(3) using the standard γ-matrix algebra

$$\Sigma(q) = \frac{N^2-1}{N}\frac{g^2}{\beta}\sum_{p_4}^{F}\int\frac{d^3p}{(2\pi)^3}\,\frac{i\gamma_\mu\hat{p}_\mu + 2m}{(\hat{p}^2+m^2)\,(p-q)^2}\,, \tag{4}$$

and then the summation is performed over the Fermi frequencies $p_4 = 2\pi T(n+1/2)$ using the well-known prescription [10]. Here $\hat{p} = \{(p_4 + i\mu), \mathbf{p}\}$ is a convenient abbreviation for vectors containing μ. All terms found are collected in a convenient form using simple algebraic transformations, and the final result is given by

$$
\Sigma(q) = -\frac{g^2(N^2-1)}{N}\int\frac{d^3p}{2(2\pi)^3}\left\{\left[\frac{1}{\epsilon_{\mathbf{p}}}\frac{n^+_{\mathbf{p}}\,[\gamma_4\epsilon_{\mathbf{p}}+(i\gamma\mathbf{p}+2m)]}{[q_4+i(\mu+\epsilon_{\mathbf{p}})\,]^2+(\mathbf{q}-\mathbf{p})^2}+\right.\right.
$$

$$
\left.\left.+\frac{n^B_{\mathbf{p}}}{|\mathbf{p}|}\frac{(|\mathbf{p}|+\mu-iq_4)\gamma_4-[i\gamma(\mathbf{q}-\mathbf{p})+2m]}{[q_4+i(\mu+|\mathbf{p}|)\,]^2+\epsilon^2_{\mathbf{p}-\mathbf{q}}}\right]-\Big[h.c.(m,\mu)\to-(m,\mu)\Big]\right\} \tag{5}
$$

where $\epsilon_{\mathbf{p}} = \sqrt{\mathbf{p}^2+m^2}$ is the bare quark self-energy; $n^B_{\mathbf{p}} = \{\exp\beta|\mathbf{p}|-1\}^{-1}$ and $n^{\pm}_{\mathbf{p}} =$ $= \{\exp\beta(\,\epsilon_{\mathbf{p}}\pm\mu)+1\}^{-1}$ are the Bose and Fermi occupation numbers, respectively.

Further it is convenient to introduce two new functions and to rewrite Eq.(5) as

$$
\Sigma(q) = i\gamma_\mu K_\mu(q) + m\ Z(q) \tag{6}
$$

where $K_\mu(q) = q_\mu a(q) + iu_\mu b(q)$, and $u_\mu = \{1,0\}$ is the unit medium vector. All functions separately depend on q_4 and $|\mathbf{q}|$, as is usual in the medium case. Equation (6) presents the one-loop decomposition of $\Sigma(q)$, which, however, is not the most general here (see [11] for details), since a number of other functions are generated only in the multi-loop calculations. Using the decomposition (6), we transform Eq.(2) to the form

$$
G(q) = \frac{-i\gamma_\mu(\hat{q}_\mu+K_\mu)+m\ (1+Z)}{(\hat{q}_\mu+K_\mu)^2\ +\ m^2\ (1+Z)^2}\,, \tag{7}
$$

which gives the correct nonperturbative structure for this function. Setting the determinant of Eq.(7) to zero, we find the dispersion relation

$$
(\hat{q}_\mu+K_\mu)^2\ +\ m^2\ (1+Z)^2 = 0, \tag{8}
$$

which determines the collective excitation spectra after the analytical continuation.

Collective excitations in the high-temperature limit. Here we use Eq.(8) to find the dispersion relation for the collective excitations of massive Dirac particles in a hot and dense quark–gluon plasma when $m \ll gT$. Different limits of this equation are discussed, and it is solved exactly for the massive fermion case with $\mu = 0$. The spectrum branches are found for all $|\mathbf{q}|$ and their limits for small and large momenta are presented explicitly. Only the case of zero damping is considered, and for this reason our analytical continuation is trivial.

Our starting point is the dispersion relation (8)

$$
[\,(iq_4-\mu)-\bar{K}_4]^2\ =\ \mathbf{q}^2\ (1+K)^2+m^2(1+Z)^2 \tag{9}
$$

with $m \neq 0$, and we use Eq.(5) to find its high-temperature expansion when $m \ll gT$. Here $K_4 = i\bar{K}_4$, and we take into account only the leading terms in T^2, with the μ/T corrections according to Eq.(9). In this case all the functions which appear in Eq.(9) can be simplified as follows:

$$
K(q_4,\mathbf{q})\ =\ \frac{I_K}{\mathbf{q}^2}\Big(\,1+\frac{\xi}{2}\ln\frac{\xi-1}{\xi+1}\Big)\ +\ I_B\,\Big(\,\xi-\frac{1}{2}(1-\xi^2)\ln\frac{\xi-1}{\xi+1}\,\Big) \tag{10}
$$

$$
-\bar{K}_4(q_4,\mathbf{q})\ =\ \frac{I_K}{2|\mathbf{q}|}\ln\frac{\xi-1}{\xi+1}+I_B\,,\qquad -Z(q_4,\mathbf{q})\ =\ 2I_Z\ +\ \frac{2I_B}{|\mathbf{q}|}\ln\frac{\xi-1}{\xi+1}\,, \tag{11}
$$

making it possible to solve Eq.(9) explicitly. Here $\xi = \omega/|\mathbf{q}|$ is a convenient variable, and the integrals are

$$I_K = \frac{g^2(N^2-1)}{N} \int_0^\infty \frac{d|\mathbf{p}|}{4\pi^2} |\mathbf{p}| \left[\frac{n_\mathbf{p}^+ + n_\mathbf{p}^-}{2} + n_\mathbf{p}^B \right], \tag{12}$$

$$I_B = -\frac{g^2(N^2-1)}{N} \int_0^\infty \frac{d|\mathbf{p}|}{8\pi^2} \frac{n_\mathbf{p}^+ - n_\mathbf{p}^-}{2}, \tag{13}$$

$$I_Z = \frac{g^2(N^2-1)}{N} \int_0^\infty \frac{d|\mathbf{p}|}{8\pi^2} \frac{n_\mathbf{p}^+ + n_\mathbf{p}^-}{2\epsilon_\mathbf{p}} . \tag{14}$$

The integral I_Z, however, has been redefined to avoid the infrared divergences which arise after the high-temperature expansion is performed for $Z(q_4, |\mathbf{q}|)$:

$$Z(q) = -\frac{g^2(N^2-1)}{N} \int \frac{d^3p}{(2\pi)^3} \left\{ \left[\frac{1}{\epsilon_\mathbf{p}} \frac{n_\mathbf{p}^+}{[q_4 + i(\mu+\epsilon_\mathbf{p})\,]^2 + (\mathbf{q}-\mathbf{p})^2} - \right.\right.$$
$$\left.\left. - \frac{n_\mathbf{p}^B}{|\mathbf{p}|} \frac{1}{[q_4 + i(\mu + |\mathbf{p}|)\,]^2 + \epsilon_{\mathbf{p}-\mathbf{q}}^2} \right] + \Big[h.c.(\mu \to -\mu) \Big] \right\} . \tag{15}$$

The last expression is extracted from Eq.(5).

Now one should plug the expressions found above into Eq.(9) and perform a number of algebraic transformations to find $\omega = \xi|\mathbf{q}|$. Here $\omega = (iq_4 - \mu)$. The result is an equation of the fourth degree with respect to $\omega(\xi)$:

$$\omega^4 \Big[\xi^2 - (1 + b(\xi)I_B)^2 \Big] + 2\omega^3\, \xi^2\, I_B + \omega^2\, \xi^2 \Big[I_B^2 - m_R^2 + 2\, d(\xi) I_K -$$
$$-2(1+b(\xi)I_B)(1+d(\xi))I_K \Big] + 2\omega\, \xi^2\, d(\xi)\, I_B\, [\, I_K + 4m_R^2\,] +$$
$$+I_K^2\, \xi^2\, [\, d(\xi)^2 - \xi^2\, (1+d(\xi))^2\,] - 16m^2\, \xi^2\, d(\xi)^2 I_B^2\, = 0 \tag{16}$$

where $m_R = m(1 - 2I_Z)$ is the renormalized fermionic mass, and the functions $d(\xi)$ and $b(\xi)$ are given by

$$d(\xi) = \frac{\xi}{2} \ln \frac{\xi - 1}{\xi + 1}; \quad b(\xi) = \xi - \frac{1}{2}(1-\xi^2) \ln \frac{\xi-1}{\xi+1} . \tag{17}$$

Since the dispersion relation obtained is very complicated, it is not solved exactly. However, in the long wavelength limit (when $\xi \to \infty$) it can be simplified as

$$\Big[\omega^2 + \omega(I_B - \eta m_R) - (I_K + 4\eta m I_B)\Big]\Big[\omega^2 + \omega(I_B + \eta m_R) - (I_K - 4\eta m I_B)\Big] = 0, \tag{18}$$

and one finds the rather simple solution [8]

$$\omega(0) = \frac{1}{2}\Big(\eta\, m_R - I_B\Big) \pm \sqrt{\frac{(\eta\, m_R - I_B)^2}{4} + (I_K + 4\eta m I_B)}\,, \tag{19}$$

which demonstrates four well-separated effective masses: two of them pertain to quasi-particle excitations and the other two to quasi-holes. Here $\eta = \pm 1$, and the parameters m and μ are nonzero.

The solutions for all $|\mathbf{q}|$ can be found within the framework of Eq.(16) if either m or μ is equal to zero. The case $m = 0$ with $\mu \neq 0$ has been recently considered in detail, and the result has the form [8]

$$E(\xi) = \mu - \frac{\xi\, I_B}{2(\xi-\eta)} \pm \sqrt{\frac{\xi^2 I_B^2}{4(\xi-\eta)^2} + I_K \xi^2 \Big(\frac{\eta}{\xi-\eta} + \frac{\eta}{2} \ln \frac{\xi-1}{\xi+1} \Big)}, \tag{20}$$

which extends the well-known result found in [2,3] to the case $\mu \neq 0$. Here we have restored the physical variable $E = ip_4$. The variable ξ runs over the range $1 < \xi < \infty$, and the long-wavelength limit corresponds to $\xi \to \infty$. For this limit one finds the very simple result

$$E(0) = \mu - \frac{I_B}{2} \pm \sqrt{\frac{I_B^2}{4} + I_K}, \tag{21}$$

which can be compared with the interpolation formula in [12].

Another case in which Eq.(16) can be solved exactly for all $|\mathbf{q}|$ is for $m \neq 0$ but $\mu = 0$. This case is the subject of the present paper and will be discussed below for $m \ll gT$. Now $I_B = 0$, and within the adopted accuracy of the calculations the solution of Eq.(16) is found to be

$$\omega_\pm(\xi)^2 = \frac{\xi^2 (\, 2I_K + m_R^2)}{2(\xi^2-1)} \pm \sqrt{\frac{\xi^4}{(\xi^2-1)^2} \Big[(b(\xi) I_K)^2 + m_R^2 (\, I_K + m_R^2/4) \Big]}\,. \tag{22}$$

These spectra are our main result. They present the collective excitations of massive Dirac particles in a hot medium for all $|\mathbf{q}|$ when $m \ll gT$. Two branches of the spectrum (when the plus sign is taken in Eq.(22)) correspond to quasi-particle excitations and the other two (when the minus sign is taken) to quasi-hole excitations. These spectrum branches differ in their asymptotic behavior and in many other properties.

The long-wavelength behavior of these spectra (when $\xi \to \infty$) has the form

$$\omega_\pm(|\mathbf{q}|)^2 = M_\pm^2 + \Big(M_\pm^2 \pm \frac{4}{9} \frac{I_K^2}{\sqrt{m_R^2 (m_R^2 + 4I_K\,)}} \Big) \frac{|\mathbf{q}|^2}{M_\pm^2} + O(|\mathbf{q}|^4) \tag{23}$$

where the squares of the effective masses are given by

$$M_\pm^2 = \frac{m_R^2}{2} + I_K \pm \sqrt{m_R^2 \Big(\frac{m_R^2}{4} + I_K \Big)}\,. \tag{24}$$

These masses are different for the four spectrum branches $M_\pm = \frac{1}{2}(\eta m_R \pm \sqrt{m_R^2 + 4I_K})$ and are in agreement with the results of [6,7]. Here $\eta = \pm 1$.

However, this is not the case when the second term in Eq.(23) is taken into account. This term is not in agreement with the one obtained in [6,7]. Although it agrees qualitatively with the result presented in [7], there is an essential difference with [6], where a linear term was mistakenly found. It is also important that the quasi-hole spectra $\omega_-(|\mathbf{q}|)^2$ are very sensitive to the choice of the parameters m, T. In many cases these spectra are monotonic functions for small $|\mathbf{q}|^2$, and the well-known minimum [2] disappears. Although this minimum always exists for massless particles, special conditions are necessary to generate it when $m \neq 0$. In the high-momentum region the asymptotic behaviors found for the quasi-particle and quasi-hole excitations are completely different. The quasi-particle branches of the spectrum are approximated as

$$\omega_+(|\mathbf{q}|)^2 = |\mathbf{q}|^2 + (2I_K + m_R^2) - \frac{I_K^2}{|\mathbf{q}|^2}\ln\frac{4|\mathbf{q}|^2}{2I_K + m_R^2}, \quad (25)$$

where the nonanalytic term is not essential. The situation is different for the quasi-hole excitations, which do not exist in the vacuum (when T and μ are equal to zero). They disappear very rapidly, and their asymptotic behavior is found to be

$$\omega_-(|\mathbf{q}|)^2 = |\mathbf{q}|^2 + 4|\mathbf{q}|^2\exp(-|\mathbf{q}|^2(2I_K + m_R^2)/I_K^2)\,. \quad (26)$$

In the high-momentum region these spectrum branches approach the line $\omega^2 = |\mathbf{q}|^2$ more quickly than (25).

Conclusion. To summarize, we have obtained and solved the one-loop dispersion relation for massive fermions at finite temperature. Our solution gives the collective Fermi excitations for all $|\mathbf{q}|$, and we have established that they have four well-separated branches: two of them represent quasi-particle excitations and the other two correspond to quasi-holes. The splitting found in the calculations demonstrates that the effective masses for all branches are different when $m \neq 0$, and these masses are always nonzero in the medium. The asymptotic behavior found for small $|\mathbf{q}|$ shows that the difference between the initially massive and massless fermions remains, although a dynamic mass is always generated and all their collective excitations are massive. For the massless fermions one finds that a spectral minimum always exists away from the point $|\mathbf{q}| = 0$, and the leading asymptotic term for small $|\mathbf{q}|$ is linear. However, this is not the case for initially massive fermions. When $m \neq 0$ the spectral minimum, as a rule, disappears as well as the linear term, and the term $|\mathbf{q}|^2$ gives the leading asymptotic behavior for small $|\mathbf{q}|$. The gauge invariance of the results found, unfortunately, is not proved, and there is no guarantee that this is indeed true. Here the situation is completely unclear, and the only known fact is that the dynamical mass for the case $m, \mu = 0$ is a gauge invariant object. All other quantities are gauge dependent, at least, within the one-loop calculations. Of course it is not ruled out that the Braaten–Pisarski resummation is necessary to improve the situation, but this question is not so evident as it is for the usual damping rate calculations.

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