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VORTEX DRAG IN QUANTUM HALL EFFECT

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A new model of momentum and electric field transfer between two adjacent 2D electron systems in the Quantum Hall Effect is proposed. The drag effect is due to momentum transfer from the vortex system of one layer to the vortex system of another layer. The remarkable result of this approach is periodical change of sign of the dragged electric field with difference between the layer filling factors.

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Drag effect in double layer two dimensional electron systems (2DES) has been the subject of intense recent interest, especially at high magnetic fields, where the Quantum Hall Effect (QHE) exists [1]. Theoretical explanations of the observed drag effect are mostly reduced to the ordinary Coulomb interaction between electrons occupying the Landau levels in the two layers [2]. These explanations are not much related to the Quantum Hall Effect. I propose a new mechanism for the momentum transfer between the layers, existing exclusively under the QHE conditions. For the simplicity I consider integer filling factors.

The model is based on a consideration of the Quantum Hall liquid as a superfluid state of the Chern-Simons charged bosons [3,4]. The ground state $\phi = \phi_1 + i\phi_2$ have a quasilong range phase correlation [5] and is uniform in the mean field approximation at "magic" filling factors $\nu_i^0 = i$, where i=1, 2... is an integer.

In accordance with this point of view on the QHE, a vortex ($H > H_i^0$) or an antivortex ($H < H_i^0$) excitations are created away from the filling factor ν_i^0 with the concentration $N_i^v = |H - H_i^0|/\Phi_0$, where H is the external magnetic field, H_i^0 is a magnetic field corresponding to the filling factor ν_i^0. The magnetic flux carried by the vortex (antivortex) is Φ_0 $(-\Phi_0)$, where $\Phi_0 = hc/e$ is the flux quantum. If the ground state carries a supercurrent $\mathbf{j}^{ext}$ the vortices experience an average force $\mathbf{F}^{ext}$, which corresponds to the force acting on the vortices in ordinary superconductor. Formally the force is a result of current-current interactions between the external supercurrent $\mathbf{j}^{ext}$ and the vortex supercurrent $\mathbf{j}^v$ [6]. Hamiltonian of the interaction is $H_{int} = \Lambda(\mathbf{j}^v\mathbf{j}^{ext})$, where Λ is an interaction constant. The negative derivative of the Hamiltonian H_{int} with respect to the vortex position is the external force $\mathbf{F}^{ext}$. The gauge invariant expression for the supercurrent of the charged bosons is the same as for ordinary superconductor, but the charge of the Cooper pair $2e$ needs to be changed by the boson charge q. The charged boson supercurrent is $\mathbf{j} = -(\Lambda c)^{-1}(\mathbf{A} - hc/2\pi q\nabla\chi)$, where χ is the phase of the boson ground state ϕ. Considering the function ϕ as one valued function, the external force F^{ext} is found to be:

$$\mathbf{F}^{ext} = \frac{\pm\Phi_0}{q_i c}[\mathbf{j}^{ext} \times \mathbf{e}_z], \tag{1}$$

where c is the light velocity, $\mathbf{e}_z$ is direction of the magnetic field $\mathbf{H}$, and the sign + (−) corresponds to the vortex (antivortex). The value q_i is the boson charge in the electron charge units: $q = q_i e$. A similar expression for the force in an external electric field $\mathbf{E}$

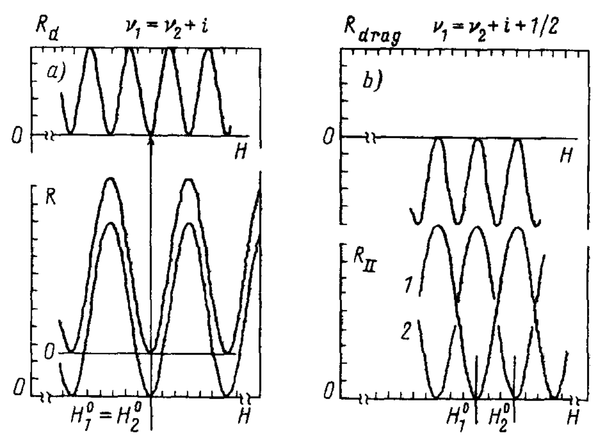
was found in a different approach [7]:

$$\mathbf{F}_2^{ext} = \frac{e^2}{hc}\Phi_0\mathbf{E}. \tag{1a}$$

Using the relation between the current and the electric field in the QHE : $j^{ext} = \sigma_{xy}E = q_i \times e^2/hE$, we obtain $F^{ext} = F_2^{ext}$.

There are several possibilities of the vortex motion in the external current $\mathbf{j}^{ext}$. A simple picture is considered here. At the temperature $T = 0\,\mathrm{K}$ all vortices are pinned by the disorder and the current $\mathbf{j}^{ext}$ flows without any dissipation. There is no momentum transfer into the vortex system at $T = 0\,\mathrm{K}$. At a finite temperature $T > 0\,\mathrm{K}$, the vortices can jump from one point to another one due to the thermal fluctuations. The external current $\mathbf{j}^{ext}$ induces an average momentum (creep) of the vortices in the direction of the force $\mathbf{F}^{ext}$. Thus, at the temperature $T > 0\,\mathrm{K}$ there is a momentum transfer from the external current to the vortex system. The average nonzero momentum of the vortices in the first layer will relax, and partially will transform into the momentum of vortices of the second layer. Actually, the most effective channel of the momentum transfer between the layers is not clear. A suitable candidate is the phonon system.

In this paper the momentum transfer between the layers is studied phenomenologically. Let us consider the momentum transfer between the vortices at some QHE resistance minimum of the first layer (H_1^0) and the vortices at some QHE resistance minimum of the second layer (H_2^0) (see figure). The average momentum of the vortices in the first (second) layer is $\mathbf{P}_1$ ($\mathbf{P}_2$). The total force from the external current $\mathbf{j}^{ext}$, acting on the vortex system (layers are squares with unit areas) is $N_i^v\mathbf{F}_i^{ext}$ (1), where i=1,2 is the layer index now.



Dependence of the drag resistance $R_{drag} = E_2/j_1^{ext}$ on magnetic field H in bilayer 2D electron system under Quantum Hall Effect conditions. Sign of the R_{drag} is positive in the case of filling factors $\nu_1 = \nu_2 + i$. The sign of the R_{drag} is negative in the case of filling factors $\nu_1 = \nu_2 + i + 1/2$, where i is an integer and 1,2 are layer indexes

Newton's equations for the vortices momentum are ($H - H_i^0 = N_i^v\Phi_0$):

$$d\mathbf{P}_1/dt = 1/(q_1c)[\mathbf{j}_1^{ext} \times (\mathbf{H} - \mathbf{H}_1^0)] - \mathbf{P}_1/\tau_1 - \mathbf{F}^{int}, \tag{2}$$

$$d\mathbf{P}_2/dt = 1/(q_2c)[\mathbf{j}_2^{ext} \times (\mathbf{H} - \mathbf{H}_2^0)] - \mathbf{P}_2/\tau_2 + \mathbf{F}^{int}, \tag{3}$$

where τ_i are the momentum relaxation rates and $\mathbf{F}^{int}$ is the interlayer drag force.

In the experiment [1], the additional electric field $\mathbf{E}_2^{ext}$ is applied to the second layer to cancel the current $\mathbf{j}_2^{ext}$. In this case the equation (3) is transformed into:

$$d\mathbf{P}_2/dt = -\mathbf{P}_2/\tau_2 + \mathbf{F}^{int}. \tag{3a}$$

At a small perturbation of the vortex distribution function the interlayer drag force is proportional to the vortex momentum $\mathbf{P}_1$. At a small value of vortex concentration N_2^v, the total drag force is proportional to the concentration N_2^v. Therefore the interlayer drag force $\mathbf{F}^{int}$ can be approximated by:

$$\mathbf{F}^{int} = \alpha N_2^v \mathbf{P}_1, \tag{4}$$

where α is a constant.

Since the interlayer drag force F^{int} is much less than the external force F^{ext}, the first one can be taken as a small perturbation in the eq.(2). From the eq.(2), (3a) and (4) the dragged momentum of the vortices $\mathbf{P}_2$ is found to be:

$$\mathbf{P}_2 = \frac{\alpha \tau_1 \tau_2}{q_1 c}[\mathbf{j}_1^{ext} \times (\mathbf{H} - \mathbf{H}_1^0)] N_2^v. \tag{5}$$

In accordance with the Maxwell electrodynamics electric field generated by the vortex movement is $\mathbf{E}_2 = 1/c[(\mathbf{H} - \mathbf{H}_2^v) \times \mathbf{V}_2]$, where $\mathbf{V}_2$ is the average vortex velocity in the second layer. The external electric field $\mathbf{E}_2^{ext}$ should cancel the electric field $\mathbf{E}_2$ to prevent the total current j_2^{ext}: $\mathbf{E}_2 + \mathbf{E}_2^{ext} = 0$.

Using the expression $\mathbf{P}_2 = m_v N_2^v \mathbf{V}_2$, where m_v is a mass of the vortex, the dragged electric field in the second layer is found to be:

$$\mathbf{E}_2 = g \mathbf{j}_1^{ext}(H - H_1^0)(H - H_2^0), \tag{6}$$

where $g = \alpha \tau_1 \tau_2 / q_1 m_v c^2$ is a constant.

1) Let's consider a case of the filling factors $\nu_1 = \nu_2 + i$ (figure a), where i is an integer (for example, equal electron concentrations $n_1^e = n_2^e$ in both layers). In this case the electric field in the second layer is

$$\mathbf{E}_2 = g \mathbf{j}_1^{ext}(H - H_1^0)^2. \tag{7}$$

In eq.(7) the drag voltage has a quadratic dependence on the magnetic field deviation from the QHE resistance minimum H_1^0. Far from the minimum H_1^0 the phase coherent ground state ϕ breaks and the vortex drag disappears. Thus each QHE minimum is accompanied by two picks of the drag voltage E_2 on different sides away from the minimum H_1^0. This behavior correlates with the experiment [1].

2) Let's make the QHE resistance minimums H_1^0 and H_2^0 in the layers not coincide with each other (figure b). In this case, in accordance with the eq.(6), the electric field E_2 changes the sign in the interval $H_1^0 < H < H_2^0$. It happens because of the antivortices ($H < H_2^0$) are dragged by the vortices ($H > H_1^0$).

If we fix the electron concentration in the first layer n_1^e and vary the concentration in the second layer n_2^e the dragged voltage sign will oscillate, due to the periodicity of the QHE conditions with the electron concentration n_e [8]. The main reason of the sign variations is the periodic change of the ground state excitations from the vortices to the antivortices with the electron concentration or with the external magnetic field.

In the conclusion, the vortex model of the drag effect in the bilayer two dimensional electron systems is proposed. Arising exclusively under the Quantum Hall Effect conditions the drag effect is induced by the momentum transfer from the vortex excitations of the ground state of the first layer to the vortex excitations of the ground state of the second layer. At equal electron concentrations in the layers the dragged voltage has the double picks structure, in accordance with the experiment [1]. For the vortex-vortex or antivortex-antivortex momentum transfer, the sign of the external electric field in the second layer is opposite to the sign of the electric field in the first layer, in accordance with the experiment [1]. For the vortex-antivortex or antivortex-vortex momentum transfer, the sign of the external electric field in the second layer is the *same* as the sign of the electric field in the first layer.

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