

# Supplemental material to the article

## Staircase structure of Shapiro steps

To demonstrate the realized states of JJs in the stack and corresponding transitions between them we simulate the average derivative of phase difference in each JJ in the stack as a function of bias current. The results at  $A = 0.05$  are shown in Fig. 1, where we indicate the values of bias current at transitions.

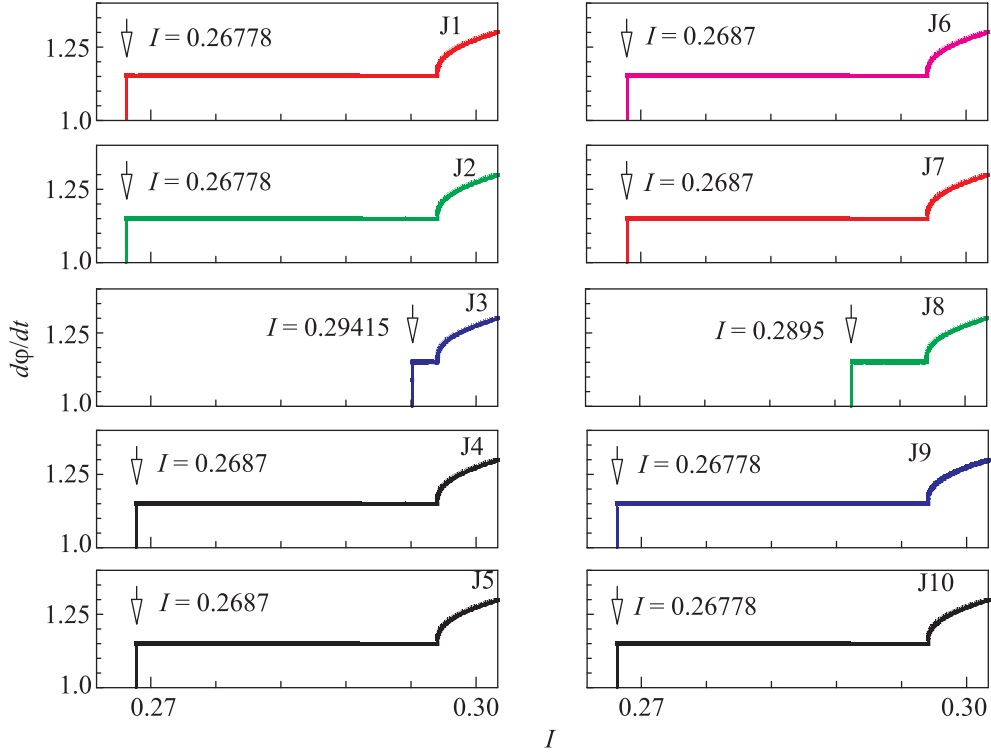


Figure 1: Average derivative of phase difference in each JJs in the stack as a function of bias current at  $A = 0.5$ . Values of current at which junction is switched from the rotating to oscillating state is written

Fig. 2 presents the  $IV$ -characteristics of all JJs in the stack at  $\omega = 1.151$  and  $A = 0.05$ . They demonstrate clearly the staircase structure of SS in each JJ. Particularly, we enlarge some parts of  $IV$ -characteristics for sixth and tenth JJs to demonstrate the different parts of SS staircase. Two first parts of staircase are actually in the chaotic state.

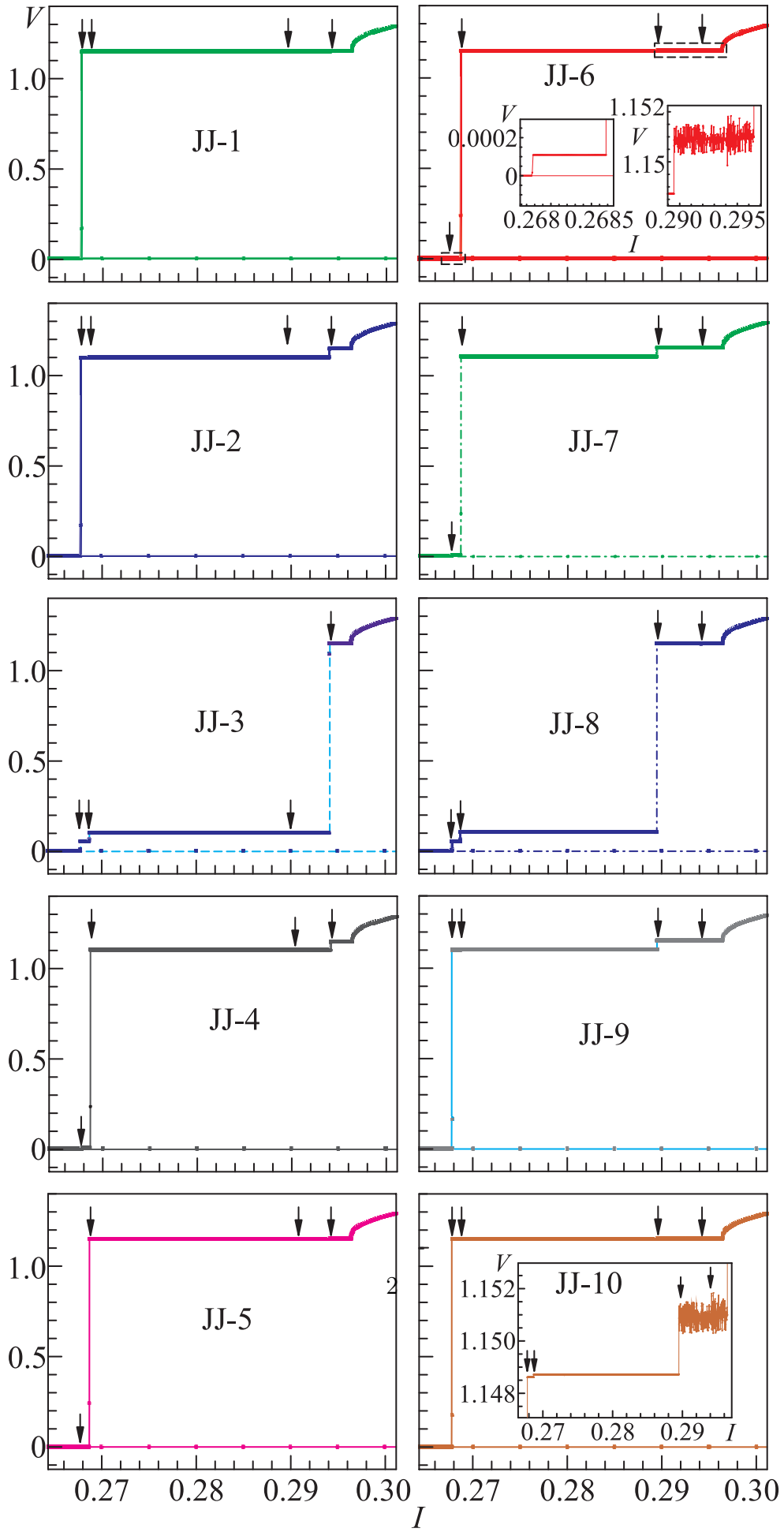


Figure 2:  $IV$ -characteristics of all JJs in the stack at  $\omega = 1.151$  and  $A = 0.05$