## Supplemental material to the article

## Spin-dependent transverse force on a vortex light beam in an inhomogeneous medium

Intensity profiles of the circularly polarized beams with different OAM. Fig. 1 displays the intensity profiles of the circularly polarized beams with different OAM in a waveguide with $\omega=7 \cdot 10^{-3} \mu \mathrm{~m}^{-}$at various distances. It is seen that the beams with antiparallel OAM and SAM can achieve tighter focal spots than those for which the signs of the helicity and the orbital angular momentum are the same. Note that the power of the longitudinal electric field $P_{e z}(z)=\iint r d r d \varphi I_{z}(r, \varphi, z)$ increases at focusing of a light beam. This indicates that the power transfer takes place from the transverse electric field component. This causes also the SAM to OAM transfer at focusing. Indeed, the significant increase of the power in the focal plane in compare to the power at the initial plane takes place, i.e. $P_{z f} / P_{z 0} \approx 5 \cdot 10^{3}$ in a given case. Note that the relative power in the longitudinal electric field component is expressed by $P_{z} / P_{\perp} \approx\left(1 / k w_{0}\right)^{2}$, i.e. the power in the longitudinal field component becomes significant if the spot size $w_{0}$ is less than the wavelength $\lambda$. In Fig. $1 \mathrm{~g}-\mathrm{j}$ the wave shapes at various distances are presented. High efficiency transfer of a strongly focused spot through optical waveguide over large distances with a period of revival is shown. The width of the incident beam oscillates with a period of $L_{0}=\pi n_{0} / \omega$ and these oscillations relax to the static value determined by the width of an incident beam. Focusing of a beam takes place at a distance of $L_{0}^{f}=\pi n_{0} /(2 \omega)$. The long-term periodic revivals of the initial width and focusing occur with a period close to $L_{\mathrm{rev}} \cong \pi m /(2 \gamma)$, where $m=1,2, \ldots, \gamma=\omega^{2} /\left(k n_{0}^{3}\right)$. Note that $L_{0} \ll L_{\mathrm{rev}}$.


Figure 1: Intensity profiles of the transverse electric field component (a,c,e,g,i) and the longitudinal electric field component (b,d,f,h,j) for the circularly polarized incident beam in the focal plane $z_{f}=331 \mu \mathrm{~m}(\mathrm{a}-\mathrm{f}), 390.1 \mathrm{~mm}$ $(\mathrm{g}, \mathrm{h})$, and $1569.2 \mathrm{~mm}(\mathrm{i}, \mathrm{j}) .(\mathrm{a}, \mathrm{b})-l=0, \sigma=1 ;(\mathrm{c}, \mathrm{d})-l=1, \sigma=1 ;(\mathrm{e}-\mathrm{j})-l=1, \sigma=-1$

