Supplemental Material to the article "On Gap Wave Vector Dependence in $Pr_{0.89}LaCe_{0.11}CuO_4$"

The expession for dynamic spin susceptibility of the electron-doped cuprates is written as follows

$$\chi_{sp}(\omega,\mathbf{q}) = \frac{\chi_q\zeta_{hJ} + (J_1K_1(4-\gamma_\mathbf{q}) - \chi_{hJ})\zeta_q}{(1+\lambda_q)\zeta_{hJ} + (\omega^2 - \Omega^2_\mathbf{q} - \frac{1}{2}\bar{J}_1h_1(4-\gamma_\mathbf{q}) - \lambda_{hJ})\zeta_q}. \tag{1}$$

The formula for dynamic charge susceptibility is

$$\chi_{ch}(\omega,\mathbf{q}) = \frac{\chi_q\zeta_h - \chi_h\zeta_q}{(1-\eta_q)\zeta_h - (\omega/2 - \eta_h)\zeta_q}. \tag{2}$$

Functions χ_q, χ_{hJ}, χ_h, ζ_q, ζ_{hJ}, ζ_h, λ_q, λ_{hJ}, η_q, η_h, $\Omega_\mathbf{q}$ are determined as:

$$\chi_q = \frac{1}{N}\sum_\mathbf{k}\chi_{kq},\ \chi_{hJ} = \frac{1}{N}\sum_\mathbf{k}M_{k,q}\chi_{kq},\ \chi_h = \frac{1}{N}\sum_\mathbf{k}\left(h_{\mathbf{k+q}} - h_\mathbf{k}\right)\chi_{kq}, \tag{3}$$

$$\zeta_q = \frac{1}{N}\sum_\mathbf{k}\zeta_{kq},\ \zeta_{hJ} = \frac{1}{N}\sum_\mathbf{k}M_{k,q}\zeta_{kq},\ \zeta_h = \frac{1}{N}\sum_\mathbf{k}\left(h_{\mathbf{k+q}} - h_\mathbf{k}\right)\zeta_{kq}, \tag{4}$$

$$\lambda_q = \frac{1}{N}\sum_\mathbf{k}\lambda_{kq},\ \lambda_{hJ} = \frac{1}{N}\sum_\mathbf{k}\left(h_{\mathbf{k+q}} - h_\mathbf{k}\right)\lambda_{kq}, \tag{5}$$

$$\eta_q = \frac{1}{N}\sum_\mathbf{k}\eta_{kq},\ \eta_h = \frac{1}{N}\sum_\mathbf{k}\left(h_{\mathbf{k+q}} - h_\mathbf{k}\right)\eta_{kq}, \tag{6}$$

$$\Omega^2_\mathbf{q} = 2J_1^2\alpha|K_1|(2-\gamma_\mathbf{q}/2)(\Delta_{sp} + 2 + \gamma_\mathbf{q}/2), \tag{7}$$

in which we used the following notations:

$$\chi_{kq} = \frac{n^h_\mathbf{k} - n^h_{\mathbf{k+q}}}{\omega + \varepsilon_\mathbf{k} - \varepsilon_{\mathbf{k+q}}}, \tag{8}$$

$$\zeta_{kq} = \frac{1}{\omega + \varepsilon_\mathbf{k} - \varepsilon_{\mathbf{k+q}}}, \tag{9}$$

$$\lambda_{kq} = \pi_{kq} + (\frac{1}{2}J'_\mathbf{q} + PT'_{\mathbf{k},\mathbf{k+q}})\chi_{kq} \tag{10}$$

$$\eta_{kq} = \frac{1}{2}\pi^h_{kq} + (\frac{1}{4}J'_\mathbf{q} + \frac{P}{2}T'_{\mathbf{k},\mathbf{k+q}} - G_\mathbf{q})\chi_{kq}, \tag{11}$$

$$\pi_{kq} = \frac{h'_{\mathbf{k+q}}Pf_\mathbf{k} - h'_\mathbf{k}Pf_{\mathbf{k+q}}}{\omega + \varepsilon_\mathbf{k} - \varepsilon_{\mathbf{k+q}}}, \tag{12}$$

$$\pi^h_{kq} = \frac{h'_\mathbf{k}n^h_\mathbf{k} - h'_{\mathbf{k+q}}n^h_{\mathbf{k+q}}}{\omega + \varepsilon_\mathbf{k} - \varepsilon_{\mathbf{k+q}}}, \tag{13}$$

$$t'_\mathbf{k} = \sum_m t_{lm}(1 - F^t_{lm})e^{-i\mathbf{k}\mathbf{R}_{lm}}, \tag{14}$$

$$T''_\mathbf{k} = \sum_{f,m\neq l}\frac{t_{lf}t_{fm}}{U}(1 - F^T_{lm})e^{-i\mathbf{k}\mathbf{R}_{lf} - i\mathbf{k}\mathbf{R}_{fm}}, \tag{15}$$

$$J'_\mathbf{q} = J_1(1 - F^J_1)\gamma_\mathbf{q}, \tag{16}$$

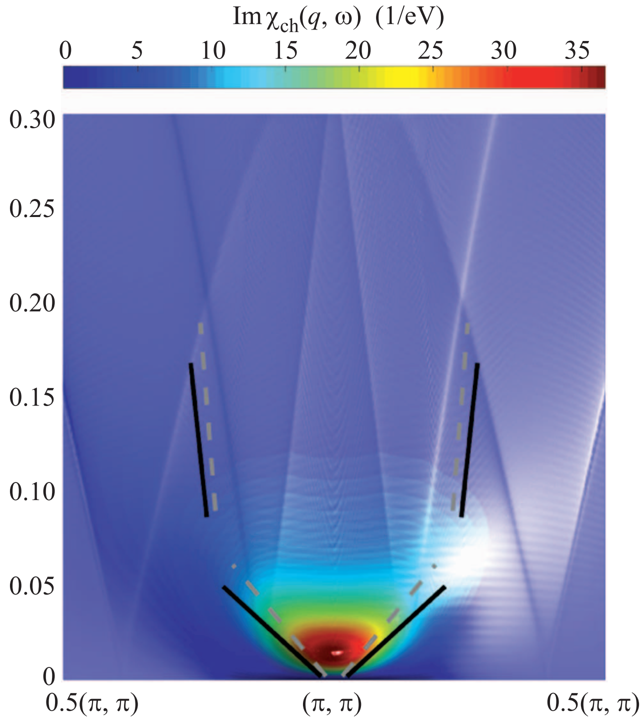

Figure 1: The imaginary part of the spin susceptibility, calculated along the diagonal of the Brillouin zone near $Q = (\pi, \pi)$. Black and gray segments correspond to the experimental results from the paper M. Fujita, K. Shigiya, J. Kaminaga, M. Nakagawa, M. Enoki, and K. Yamada, J. Phys. Soc. Jpn. **80**, SB029 (2011).

$$T'_{\mathbf{k},\mathbf{k+q}} = \frac{t_1^2}{U}(1 - F_1^T)\gamma_{\mathbf{2k+q}}, \tag{17}$$

$$G_{\mathbf{q}} = \frac{2\pi d e^2}{\varepsilon_\perp a^2 \sqrt{A^2 - 1}}, \tag{18}$$

$$A = \left(2\frac{\varepsilon_\parallel}{\varepsilon_\perp} \frac{\sin^2(q_x a/2)}{(a/d)^2} + 2\frac{\varepsilon_\parallel}{\varepsilon_\perp} \frac{\sin^2(q_y a/2)}{(a/d)^2} + 1 \right), \tag{19}$$

$$M_{k,q} = \frac{1}{2}(\bar{J}_{\mathbf{k+q}} - \bar{J}_{\mathbf{k}} - 2\omega)(h_{\mathbf{k+q}} - h_{\mathbf{k}}) - J_1 \frac{t_1^2}{U} L_{k,q}, \tag{20}$$

$$L_{k,q} = K_1(\gamma_{\mathbf{q}} - 4)\gamma_{\mathbf{2k+q}} + \frac{1-\delta_0}{2}(\gamma_{\mathbf{2k+2q}} + \gamma_{\mathbf{2k}} - 2\gamma_{\mathbf{2k+q}}). \tag{21}$$

$n^h_{\mathbf{k}}$ denotes the number of quasi-particls in the hole representation; $n^h_{\mathbf{k}} = P f^h_{\mathbf{k}}$, $P = (1+\delta_0)/2$, $f^h_{\mathbf{k}} = 1/(1 + e^{-\varepsilon_{\mathbf{k}}/k_B T})$.

The energy of quasi-particles, is written as

$$\varepsilon_{\mathbf{k}} = -\mu + 2t(\cos k_x a + \cos k_y a) + 4t' \cos k_x a \cos k_y a + 2t''(\cos 2k_x a + \cos 2k_y a). \tag{22}$$

The effective hopping parameters of quasi-particles "dressed with charge and spin correlations" are expressed in terms of the bare hopping parameters of electrons between the first (t_1), second (t_2) and third neighbors (t_3) in the following way

$$t = t_1 \left(P + \frac{1 + 2F_1^t}{4P} K_1 \right) + \left(J_1 \frac{F_1^J}{2P} + 3\frac{t_1^2}{U}(1 + F_1^T) \right) \langle X_0^{0,\sigma} X_1^{\sigma,0} \rangle, \tag{23}$$

$$t' = t_2 P + 2\frac{t_1^2}{U}\left(\frac{2+F_1^T}{2}K_1 - \frac{1-\delta_0}{2}P\right), \tag{24}$$

$$t'' = t_3 P + \frac{t_1^2}{U}\left(\frac{2+F_1^T}{2}K_1 - \frac{1-\delta_0}{2}P\right). \tag{25}$$

$K_1 = 4\langle S_0^z S_1^z\rangle$ denotes spin-spin correlation function, which are estimated self-consistently through dynamic spin susceptibility.

The parameters were taken as: $t = 0.2$ eV, $t' = -0.4t$, $t'' = 0.1t$, $\mu = -0.4t$. The exchange integral between the nearest neighbors is $J_1 = 0.11$ eV. Projective parameters are $F_1^J = F_1^T = 0.7$. The spin-spin correlation function for the nearest neighbors is $K_1 = -0.22$