Supplemental Material to the article

"Absorption of Electromagnetic Waves by Plasma Oscillations in Infinite 2D Electron Gas in Magnetic Field"

1. Calculation of optical coefficients. Let us consider a linearly polarized electromagnetic wave frequency ω incident from a vacuum at an angle θ to the normal of the 2D electron system. Axis z is perpendicular to the 2D system. For s-polarization, the incident electric field vector $\mathbf{E}^i = (E_x^i, 0, 0)$ is perpendicular to the plane of incidence, and the vector of magnetic field lies in the plane of incidence and has the form $\mathbf{H}^i = (0, -E_x^i \cos\theta, -E_x^i \sin\theta)$. All components of vectors are proportional to $e^{i\mathbf{kr}-i\omega t}$, where $\mathbf{k}$ is the wave vector.

The electromagnetic fields of the reflected and transmitted waves, marked by the index r and t, respectively, have the form $\mathbf{E}^r = (E_x^r, E_y^r, -E_y^r \tan\theta)$, $\mathbf{H}^r = (-E_y^r/\cos\theta, E_x^r \cos\theta, -E_x^r \sin\theta)$, $\mathbf{E}^t = (E_x^t, E_y^t, -E_y^t \tan\theta)$, $\mathbf{H}^t = (E_y^t/\cos\theta, -E_x^t \cos\theta, -E_x^t \sin\theta)$.

Interface conditions for transmitted, reflected and incident waves at $z = 0$:

$$\begin{cases} \mathbf{E}_{||}^{+} = \mathbf{E}_{||}^{-} \\ H_y^- - H_y^+ = \frac{4\pi}{c} j_x = \frac{4\pi}{c}\left(\sigma_{xx} E_x^- + \sigma_{xy} E_y^-\right) \\ H_x^+ - H_x^- = \frac{4\pi}{c} j_y = \frac{4\pi}{c}\left(\sigma_{yx} E_x^- + \sigma_{yy} E_y^-\right) \end{cases}, \tag{1}$$

where $\mathbf{E}_{||}^{+}$ and $\mathbf{E}_{||}^{-}$ are the projection of vectors $\mathbf{E}^i + \mathbf{E}^r$ and $\mathbf{E}^t$ on the plane $z = 0$ respectively, σ_{ij} is the components of the conductivity tensor. The Amplitude reflection and transmission coefficients

$$r_x = \frac{E_x^r}{E_x^i}, \quad r_y = \frac{E_{||}^r}{E_x^i} = \frac{E_y^r}{E_x^i \cos\theta}, \tag{2}$$

$$t_x = \frac{E_x^t}{E_x^i}, \quad t_y = \frac{E_{||}^t}{E_x^i} = \frac{E_y^t}{E_x^i \cos\theta}, \tag{3}$$

is obtained from Eq. (1):

$$r_x = -\frac{\alpha_{xx}\alpha_{yy} + \alpha_{xx}\sec\theta - \alpha_{xy}\alpha_{yx}}{1 + \alpha_{xx}\alpha_{yy} + \alpha_{xx}\sec\theta + \alpha_{yy}\cos\theta - \alpha_{xy}\alpha_{yx}}, \tag{4}$$

$$r_y = -\frac{\alpha_{yx}}{1 + \alpha_{xx}\alpha_{yy} + \alpha_{xx}\sec\theta + \alpha_{yy}\cos\theta - \alpha_{xy}\alpha_{yx}}, \tag{5}$$

$$t_x = \frac{1 + \alpha_{yy}\cos\theta}{1 + \alpha_{xx}\alpha_{yy} + \alpha_{xx}\sec\theta + \alpha_{yy}\cos\theta - \alpha_{xy}\alpha_{yx}}, \tag{6}$$

$$t_y = r_y, \tag{7}$$

where we denote $\alpha_{ij} = \frac{2\pi\sigma_{ij}}{c}$.

Zeros of coefficients (4)–(7) would have coincided with the dispersion equation, that is describe magnetoplasma oscillation [10] if $\cos\theta$ was equal to $i\beta c/\omega$, where $\beta = \sqrt{q^2 - \omega^2/c^2}$, and q is the magnetoplasma wave vector. However, in [10] $Re(\beta) \geq 0$ since electromagnetic fields decay from the 2D electron system. This leads to the condition $Im(\beta) \geq 0$ for all magnetoplasma waves in [10]. It means that $i\beta c/\omega < 0$ and the expression can not determine any real cosine of angle of incidence. Thus, the magnetoplasmon dispersion equation does not determine poles of optical coefficients.

Power coefficients is squared amplitude coefficients: reflectivity $R = |r|^2 = |r_x|^2 + |r_y|^2$, transmissivity $T = |t|^2 = |t_x|^2 + |t_y|^2$, absorptivity $A = 1 - R - T$.

2. Direction of the Poynting vector of the magnetoplasmon. First, we calculate electric and magnetic fields of the the magnetoplasmon, predicted in [10]. If plasmon propagates along the y-axis, see inset in the Fig. 1, then

$$E_y = \frac{i\alpha_{xy}\beta}{i\alpha_{xx}\beta + \Omega} E_x \tag{8}$$

$$E_z = -\frac{\alpha_{xy}\beta}{i\alpha_{xx}\beta + \Omega} E_x \tag{9}$$

$$H_x = -\frac{\alpha_{xy}\Omega}{i\alpha_{xx}\beta+\Omega}E_x \tag{10}$$

$$H_y = -\frac{\alpha_{xx}\Omega+i\beta\left(\alpha_{xx}^2+\alpha_{xy}^2\right)}{i\alpha_{xx}\beta+\Omega}E_x \tag{11}$$

$$H_z = -i\frac{Q}{\beta}\cdot\frac{\alpha_{xx}\Omega+i\beta\left(\alpha_{xx}^2+\alpha_{xy}^2\right)}{i\alpha_{xx}\beta+\Omega}E_x, \tag{12}$$

where $\beta = \sqrt{Q^2-\Omega^2}$, $Q = qc\tau$ is the dimensionless plasmon wave vector.

For low frequency $\Omega \ll 1$ and high magnetic field $\Omega_c \gg 1$ one can obtain the dispersion of the magnetoplasmon:

$$\Omega = Q + \frac{\alpha^2 Q}{2\Omega_c^4}\left(1-Q^2\right) - i\frac{\alpha^2 Q}{\Omega_c^4} + o\left(\frac{1}{\Omega_c^4}\right), \tag{13}$$

and find the Poyting vector $\mathbf{S} = c\cdot Re\left[\mathbf{E}\times\mathbf{H}^*\right]/8\pi$.

The angle between the poyting vector and the normal to the 2D electron system

$$\cos\theta = \frac{|S_z|}{S} = \frac{\alpha}{\Omega_c^2} + o\left(\frac{1}{\Omega_c^4}\right) \tag{14}$$

is exactly the same angle at which the maximum absorption of electromagnetic waves is observed.