Supplemental Material to the article

“Magnetoabsorption in HgCdTe/CdHgTe Quantum Wells in Tilted Magnetic Fields”

1. Calculation of Landau levels in tilted magnetic field. The total Hamiltonian is sum of 3 terms: Kane Hamiltonian, the deformation Hamiltonian and one describing the symmetry lowering due to anisotropy of the chemical bonds at the quantum well (QW) interfaces (interface inversion asymmetry, IIA). The last two terms do not depend on the magnetic field. The explicit form of the deformational Hamiltonian for (013) oriented HgCdTe QW is given in [3] and that Hamiltonian describing IIA is given in [26].

Choose the vector potential as follows: only the components in the plane of the quantum well are nonzero:

$$A_x = A'_x - B_y z, A_y = A'_y + B_z z, A_z = 0.$$

Components with primes describe the magnetic field along z axis:

$$B_z = \frac{\partial A'_y}{\partial x} - \frac{\partial A'_x}{\partial y}.$$

Let us introduce creation and annihilation operators:

$$a^+ = \frac{\lambda}{\sqrt{2}}(k_x + ik_y) = \frac{\lambda}{\sqrt{2}}k_+, a = \frac{\lambda}{\sqrt{2}}(k_x - ik_y) = \frac{\lambda}{\sqrt{2}}k_-,$$

where

$$k_x = -i\frac{\partial}{\partial x} + \frac{e}{\hbar c}A'_x, k_y = -i\frac{\partial}{\partial y} + \frac{e}{\hbar c}A'_y, \lambda = \sqrt{\frac{\hbar c}{|eB_z|}}.$$

Here λ is the magnetic length, $-e$ is the electron charge. The operators k_x and k_y satisfy the following commutation relation $[k_x, k_y] = -i\frac{eB_z}{\hbar c}$. Therefore the commutator of a and a^+ can be expressed as:

$$[a, a^+] = \frac{eB_z}{|eB_z|}.$$

For $eB_z > 0$ ($B_z > 0$) $[a, a^+] = 1$.

As easy to show in the presence of the magnetic fields the following substitutions can be done:

$$k_+ \to \frac{\sqrt{2}}{\lambda}a^+ + i\frac{e}{c\hbar}zB_+, k_- \to \frac{\sqrt{2}}{\lambda}a - i\frac{e}{c\hbar}zB_-.$$

Just as in the case of the normal magnetic field it is suitable to divide the total Hamiltonian including deformational, Zeeman and IIA terms into axially symmetric and anisotropic parts. Without the Zeeman term the axially symmetric part of the Hamiltonian has the following form:

$$H_s = \begin{pmatrix}
T & 0 & -\frac{P}{\lambda}a^+ & \sqrt{\frac{2}{3}}Pk_z & \frac{Pa}{\sqrt{3}\lambda} & 0 & -\frac{P}{\sqrt{3}}k_z & -\sqrt{\frac{2}{3}}\frac{P}{\lambda}a \\
0 & T & 0 & -\frac{Pa^+}{\sqrt{3}\lambda} & \sqrt{\frac{2}{3}}Pk_z & \frac{P}{\lambda}a & -\sqrt{\frac{2}{3}}\frac{P}{\lambda}a^+ & \frac{P}{\sqrt{3}}k_z \\
-\frac{P}{\lambda}a & 0 & U+V & S+s & R & 0 & -\frac{S+s}{\sqrt{2}} & -\sqrt{2}R \\
\sqrt{\frac{2}{3}}Pk_z & -\frac{Pa}{\sqrt{3}\lambda} & S^+ + s^+ & U-V & C & R & \sqrt{2}V & \sqrt{\frac{3}{2}}Z \\
\frac{Pa^+}{\sqrt{3}\lambda} & \sqrt{\frac{2}{3}}Pk_z & R^+ & C^+ & U-V & -S+s & \sqrt{\frac{3}{2}}z & -\sqrt{2}V \\
0 & \frac{P}{\lambda}a^+ & 0 & R^+ & -S^+ + s^+ & U+V & \sqrt{2}R^+ & -\frac{S^+ - s^+}{\sqrt{2}} \\
-\frac{P}{\sqrt{3}}k_z & -\sqrt{\frac{2}{3}}\frac{P}{\lambda}a & -\frac{S^+ + s^+}{\sqrt{2}} & \sqrt{2}V & -\sqrt{\frac{3}{2}}z^+ & \sqrt{2}R & U-\Delta & C \\
-\sqrt{\frac{2}{3}}\frac{P}{\lambda}a^+ & \frac{P}{\sqrt{3}}k_z & -\sqrt{2}R^+ & -\sqrt{\frac{3}{2}}Z^+ & -\sqrt{2}V & -\frac{S-s}{\sqrt{2}} & C^+ & U-\Delta
\end{pmatrix},$$

where

$$T = E_c + \frac{\hbar^2}{2m_0}(k_z(2F+1)k_z) + (2F+1)\hbar\omega_c(a^+a+\frac{1}{2}) + \frac{\hbar^2(2F+1)}{2m_0}(\frac{ez}{c\hbar})^2({B_x}^2+{B_y}^2),$$

$$U = E_v - \frac{\hbar^2}{2m_0}(k_z\gamma_1 k_z) - \hbar\omega_c\gamma_1(a^+a+\frac{1}{2}) - \frac{\hbar^2\gamma_1}{2m_0}(\frac{ez}{c\hbar})^2({B_x}^2+{B_y}^2),$$

$$V = -\frac{\hbar^2}{2m_0}(0.54k_z(\gamma_2-\gamma_3)k_z - 2k_z\gamma_2 k_z) + \frac{\hbar\omega_c(0.27(\gamma_2-\gamma_3)-2\gamma_2)}{2}(a^+a+\frac{1}{2}) - \frac{\hbar^2\gamma_2}{2m_0}(\frac{ez}{c\hbar})^2({B_x}^2+{B_y}^2)+$$

$$+0.27\frac{\hbar^2(\gamma_2-\gamma_3)}{2m_0}(\frac{ez}{c\hbar})^2({B_x}^2) - 0.72\frac{\hbar e B_x}{2m_0 c\hbar}\{(\gamma_2-\gamma_3)z, k_z\},$$

$$S = \frac{\hbar^2\sqrt{6}}{2m_0\lambda}(\{\gamma_3, k_z\} + 0.18\{(\gamma_2-\gamma_3), k_z\})a + \frac{\hbar e\sqrt{6}}{m_0\lambda c}0.24(\gamma_2-\gamma_3)B_x z a,$$

$$s = \frac{\hbar^2}{2m_0\lambda}a[\kappa, k_z],$$

$$R = \hbar\omega_c\sqrt{3}(\gamma_2 - 0.545(\gamma_2-\gamma_3))a^2,$$

$$Z = \frac{\hbar^2\sqrt{6}}{2m_0\lambda}(0.18\{(\gamma_2-\gamma_3), k_z\} + \{\gamma_3, k_z\} - \frac{1}{3}[\kappa, k_z])a + 0.24\frac{\hbar e\sqrt{6}}{m_0\lambda c}(\gamma_2-\gamma_3)zB_x a,$$

$$z = \frac{\hbar^2\sqrt{6}}{2m_0\lambda}(\{\gamma_3, k_z\} + 0.18\{(\gamma_2-\gamma_3), k_z\} + -\frac{1}{3}[\kappa, k_z])a^+ + 0.24\frac{\hbar e\sqrt{6}}{m_0\lambda c}(\gamma_2-\gamma_3)zB_x a^+,$$

$$C = \frac{\hbar^2\sqrt{2}}{m_0\lambda}[\kappa k_z]a.$$

References (as in the paper)

3. M. Zholudev, F. Teppe, M. Orlita et al. (Collaboration), Phys. Rev. B **86**, 205420 (2012).

26. G.M. Minkov, V.Ya. Aleshkin, O.E. Rut, A.A. Sherstobitov, A.V. Germanenko, S.A. Dvoretski, and N.N. Mikhailov, Phys. Rev. B **96**, 035310 (2017).