Supplemental Material to the article
"Resonances of the Faraday effect in nanostructured iron-garnet films"

Let us consider a waveguide layer with the dielectric permittivity ε_{eff} and thickness h_{eff} surrounded by air and gadolinium gallium garnet (GGG). The coordinate axes are chosen in such a way that x-axis is directed along the modes propagation and z-axis is perpendicular to the interfaces of the waveguide layer. If the layer is magnetized along z-axis the electromagnetic field inside the layer has the following form:

$$\mathbf{E}(x,z) = (K_1\mathbf{e}_2^{(1)}\exp(i\gamma_a z) + K_2\mathbf{e}_2^{(2)}\exp(-i\gamma_a z) + K_3\mathbf{e}_2^{(3)}\exp(i\gamma_b z) + K_4\mathbf{e}_2^{(4)}\exp(-i\gamma_b z)\exp(i[\beta x - \omega t])), \quad \text{(S1)}$$

where ω is the frequency and $\mathbf{e}_2^{(l)}$ is the unitary electric field vector for certain polarization denoted by "a" or "b". Polarization and $\gamma_{a,b}$ are found from the Fresnel equation:

$$(\mathbf{k}^{(l)})^2\mathbf{e}_2^{(l)} - \mathbf{k}^{(l)}(\mathbf{k}^{(l)}\mathbf{e}_2^{(l)}) = \frac{\omega^2}{c^2}\hat{\varepsilon}\mathbf{e}_2^{(l)}, \quad \text{(S2)}$$

where $\mathbf{k}^{(l)}$ is the mode's wavevector, $\mathbf{k}^{(1,2)} = \{\beta; 0; \pm\gamma_a\}$ and $\mathbf{k}^{(3,4)} = \{\beta; 0; \pm\gamma_b\}$, and $\hat{\varepsilon}$ is the dielectric tensor of the layer, c is the vacuum light velocity. In surrounding media which are optically isotropic the electromagnetic field is decomposed into TE and TM components:

$$\mathbf{E}(x,z) = (A_j\mathbf{e}_j^{(\text{TE})} + B_j e_j^{(\text{TM})})\exp(-\gamma_j|z|)\exp(i[\beta x - \omega t]), \quad \text{(S3)}$$

where $j = 1$ and $j = 3$ for air and GGG, respectively, $\gamma_{1,3} = \sqrt{\beta^2 - \varepsilon_{1,3}\frac{\omega^2}{c^2}}$, ε_j is the corresponding dielectric permittivity. Applying the boundary conditions for the tangential components of the electric and magnetic field and taking into account Eqs. (S1) and (S3) one can find that the waveguide modes are not purely TM- or TE-polarized but have additional components proportional to gyration g. It can be written in the following form:

$E_y^{(\text{TM})}(x,z) = K(\beta,z)H_y^{(\text{TM})}(x,z)$ for quasi-TM modes,

$E_y^{(\text{TE})}(x,z) = L(\beta,z)H_y^{(\text{TE})}(x,z)$ for quasi-TE modes.

Here linear in g approximation is applied, provided that for iron garnets usually $g \ll \varepsilon$. The formulas for functions $K(\beta,z)$ and $L(\beta,z)$ are cumbersome, so we present them only for $z = 0$, which corresponds to the interface between the waveguide layer and GGG:

$$K(\beta, z=0) = i\frac{\left(\varepsilon_1 + \varepsilon_{\text{eff}}\dfrac{\gamma_1}{\gamma_3}\right)k_0 h_{\text{eff}}\sin\chi - k_0\left(\varepsilon_{\text{eff}}\dfrac{\gamma_1}{\gamma_2} - \varepsilon_1\dfrac{\gamma_2}{\gamma_3}\right)h_{\text{eff}}\cos\chi + \dfrac{k_0}{\gamma_2}\left(\varepsilon_{\text{eff}}\dfrac{\gamma_1}{\gamma_2} + \varepsilon_1\dfrac{\gamma_2}{\gamma_1}\right)\sin\chi}{2\varepsilon_1\varepsilon_{\text{eff}}\left(\left(1 + \dfrac{\gamma_1}{\gamma_3}\right)\cos\chi + \left(\dfrac{\gamma_1}{\gamma_2} - \dfrac{\gamma_2}{\gamma_3}\right)\sin\chi\right)},$$

$$L(\beta, z=0) = -i\frac{\left(\varepsilon_1 + \varepsilon_{\text{eff}}\dfrac{\gamma_1}{\gamma_3}\right)k_0 h_{\text{eff}}\sin\chi - k_0\left(\varepsilon_{\text{eff}}\dfrac{\gamma_1}{\gamma_2} - \varepsilon_1\dfrac{\gamma_2}{\gamma_3}\right)h_{\text{eff}}\cos\chi - \dfrac{k_0}{\gamma_2}\left(\varepsilon_{\text{eff}}\dfrac{\gamma_1}{\gamma_2} + \varepsilon_1\dfrac{\gamma_2}{\gamma_1}\right)\sin\chi}{2\varepsilon_1\varepsilon_{\text{eff}}\left(\left(1 + \dfrac{\varepsilon_3\gamma_1}{\varepsilon_1\gamma_3}\right)\cos\chi + \left(\dfrac{\varepsilon_{\text{eff}}\gamma_1}{\varepsilon_1\gamma_2} - \dfrac{\varepsilon_3\gamma_2}{\varepsilon_{\text{eff}}\gamma_3}\right)\sin\chi\right)}, \quad \text{(S4)}$$

where $\gamma_2 = \sqrt{\varepsilon_{\text{eff}}\frac{\omega^2}{c^2} - \beta^2}$ and $\chi = \gamma_2 h_{\text{eff}}$, ε_{eff} and h_{eff} are the dielectric permittivity and thickness of the waveguide layer.

It follows from Eqs. (S4) that functions $K(\beta,z)$ and $L(\beta,z)$ have different sign and they have values of the same order: $K(\beta,z) \cong -L(\beta,z)$.