Supplementary Material to the article

"Origin of structure inversion asymmetry in double HgTe quantum wells"

A. Approximation of magnetoabsorption lines with multiple Lorentzians. Each magnetoabsorption spectrum was fitted using the sum of several Lorentzians (Fig. S1). The position (E), the full width at half maximum (FWHM), and the area (A) of each spectral feature was determined.

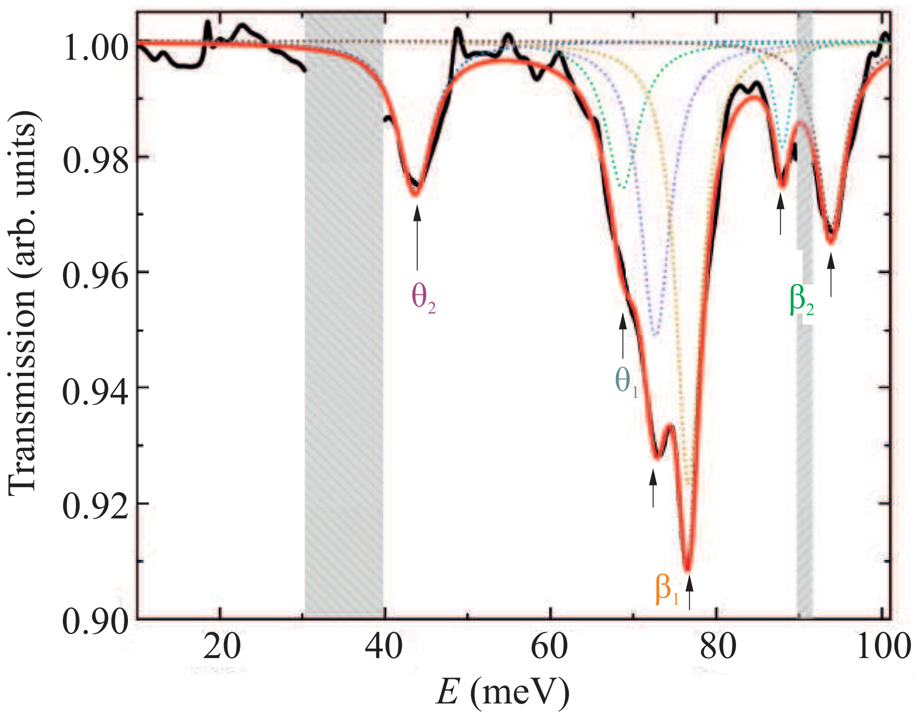

Fig. S1. Magnetoabsorption spectrum of sample A in a magnetic field of 11 T (black curve) and its approximation with multiple Lorentzians (red curve). Feature notations correspond to [1]. The shaded areas are the reststrahlen ?? band of the GaAs substrate (left) and the opacity region of Mylar multilayer beamsplitter (right)

Table S1. The approximation parameters for the magnetoabsorption spectrum shown in Fig. S1. The numbering of lines in the Table corresponds to the sequential numbering of the Lorentzians in Fig. 1 from left to right

Feature	θ_1	θ_2	β_1		β_2	
#	1	2	3	4	5	6
E, meV	43.60 ± 0.04	68.65 ± 0.09	$\mathbf{72.67} \pm 0.04$	$\mathbf{76.66} \pm 0.02$	$\mathbf{87.95} \pm 0.04$	$\mathbf{93.86} \pm 0.03$
FWHM, meV	5.04 ± 0.13	4.77 ± 0.22	4.39 ± 0.22	3.70 ± 0.06	2.34 ± 0.14	4.06 ± 0.11
A	0.20 ± 0.004	0.19 ± 0.015	0.35 ± 0.02	0.45 ± 0.011	0.067 ± 0.003	0.207 ± 0.004

2. Magnetic field dependence of the splittings of β_1 and β_2 lines. After approximating the magnetoabsorption spectra and determining the position of all spectral features, the ΔE splittings for the β_1 and β_2 lines were founded. Also their dependence on the magnetic field was obtained (Fig. S2). The error in determining the values of ΔE is upper estimated as 0.7 meV ($\sqrt{2}$ of half of the spectral resolution).

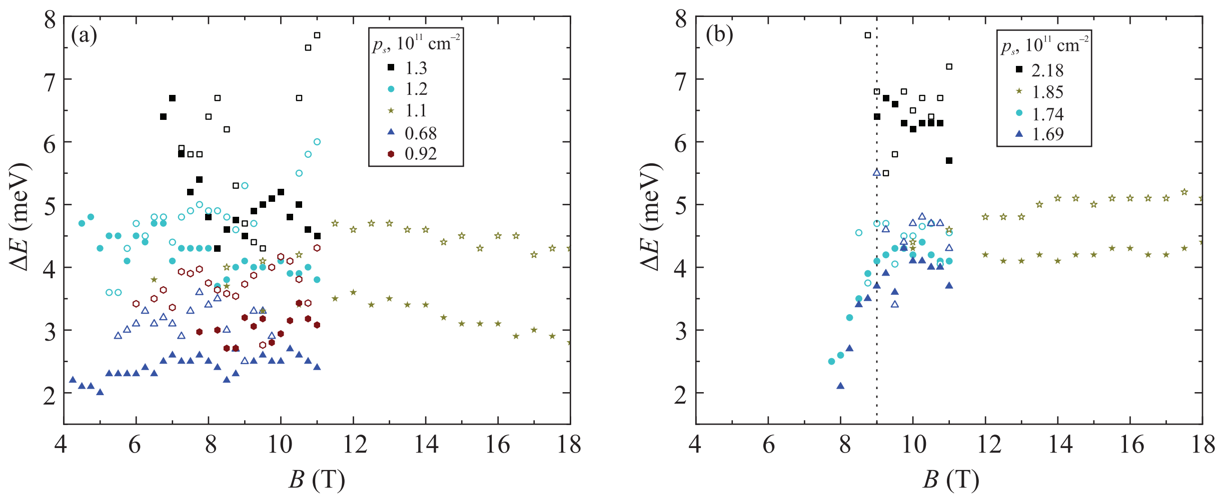

Fig. S2. Dependences of the ΔE splittings on the magnetic field in samples A (a) and B (b) for the β_1 (solid symbols) and β_2 (open symbols) lines at various hole concentrations. Data obtained at $B > 18$ T are not shown for clarity

As one can see from the Fig. S2, no significant dependence of ΔE on the magnetic field is observed, except for a clear decrease in splitting with decreasing magnetic field at $B < 9$ T for sample B (Fig. S3b). The latter is associated with the manifestation of anticrossing of 0 and -2 "zero-mode" Landau levels in the HgTe QW (see, for example [2–4]) and the change in their energies. Thus, for the sample B, in further analysis, we took into account only the data at $B \geq 9$ T.

Since no pronounced dependence of the ΔE splittings on the magnetic field was observed, all the values were averaged for a given concentration of charge carriers and for the specified transition (Fig. S3).

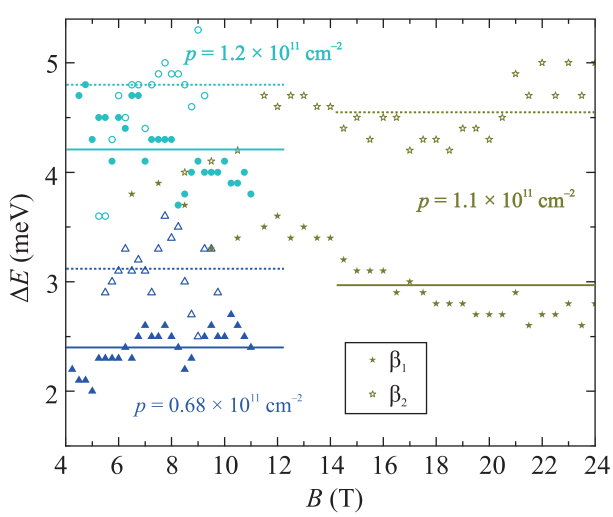

Fig. S3. Dependences of the ΔE splittings on the magnetic field in sample A for the β_1 and β_2 lines at various hole concentrations. The horizontal lines indicate the average values of the splittings. Data obtained at $B > 24$ T are not shown for clarity

3. Dependences of the ΔE splittings on the concentration of charge carriers. The built-in electric field in a system of double quantum wells (DQW) of the same thickness leads to the removal of the degeneracy for the Landau levels -2 and to the emerging of energy difference ΔE between them. Assuming that all charge carriers in the DQW "came" from the surface, an increase in the charge carrier concentration means an increase in the built-in electric field and, accordingly, in the ΔE splitting (Fig. S4). In this case, if the type of conductivity is hole, then the concentration dependence of the splitting for sample B has an inflection associated with the filling of the side maximum in the valence band (solid curve in Fig. S4). In the case of the electron type of conductivity since no side maxima in the conduction band, no singularities are observed (dashed curve in Fig. S4).

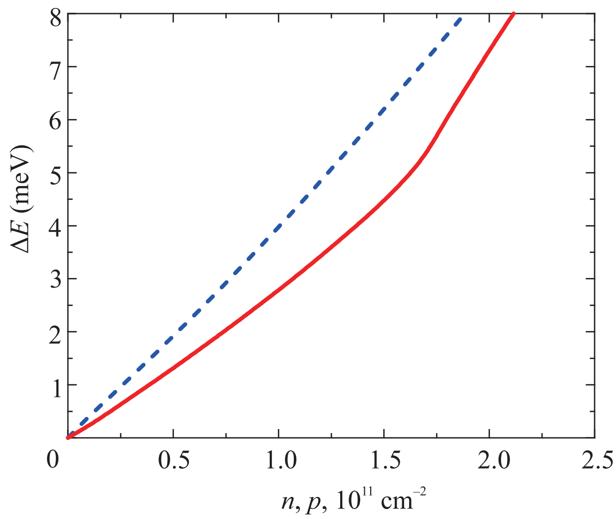

Fig. S4. The calculated dependences of the ΔE splitting between $H1$ and $H2$ subbands at $k = 0$ on the concentration of charge carriers for the sample B under the assumption that all charge carriers "came" from the surface and quantum wells thicknesses are equal. The solid curve corresponds to the hole type of conductivity, the dashed curve corresponds to the electron one

1. L. S. Bovkun, A. V. Ikonnikov, V. Ya. Aleshkin, K. V. Maremyanin, N. N. Mikhailov, S. A. Dvoretskii, S. S. Krishtopenko, F. Teppe, B. A. Piot, M. Potemski, M. Orlita, and V. I. Gavrilenko, Opto-Electronics Review **27**, 213 (2019).
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