Supplementary Material to the article

"New Objects in Scattering Theory with Symmetries"

Here we present the terms needed to verify equation

$$[V_{\text{ind}}, S_0] + V_{\text{ind}} S_{\text{L}} - S_{\text{R}} V_{\text{ind}} = 0 \tag{S1}$$

up to λ^3 for the double δ-potential on S^1_R.

As in the main text, terms proportional to $\lambda^\alpha R^{-\beta}$ are denoted with (α, β).

V_{ind} up to λ^3.

$$V_{\text{ind}}^{(1,1)} = \begin{pmatrix} V_{n_0,n_0} & V_{n_0,-n_0} \\ V_{-n_0,n_0} & V_{-n_0,-n_0} \end{pmatrix} = (-1)^F \frac{\lambda \sin 2a\kappa}{\pi R} \sigma_2. \tag{S2}$$

$$V_{\text{ind}}^{(2,1)} = -\frac{\lambda^2 \sin 2a\kappa}{2\pi R \kappa} (\cos 2a\kappa + \sigma_1). \tag{S3}$$

$$V_{\text{ind}}^{(2,2)} = \frac{\lambda^2 a \sin 2a\kappa}{\pi^2 R^2 \kappa} \left(\cos 2a\kappa - \frac{\sin 2a\kappa}{4a\kappa} + \sigma_1 \right). \tag{S4}$$

$$V_{\text{ind}}^{(3,1)} = -(-1)^F \frac{\lambda^3 \sin^3 2a\kappa}{4\pi R \kappa^2} \sigma_2. \tag{S5}$$

$$V_{\text{ind}}^{(3,2)} = (-1)^F \frac{\lambda^3 (\cos 2a\kappa + 4a\kappa \sin 2a\kappa) \sin^2 2a\kappa}{4\pi^2 R^2 \kappa^3} \sigma_2. \tag{S6}$$

$$V_{\text{ind}}^{(3,3)} = -(-1)^F \frac{\lambda^3 \left(8a\kappa \cos 2a\kappa + (16a^2\kappa^2 - 1) \sin 2a\kappa\right) \sin^2 2a\kappa}{16\pi^3 R^3 \kappa^4} \sigma_2. \tag{S7}$$

S_{L} up to λ^2.

$$S_{\text{L}}^{(1,1)} = \psi \frac{a\lambda}{\pi R} \left(1 + \frac{\sin 2a\kappa}{2a\kappa} \sigma_1 \right). \tag{S8}$$

$$S_{\text{L}}^{(2,1)} = \psi \frac{\mathrm{i}\lambda^2 a}{2\pi R \kappa} \Big((1 - \tfrac{\sin 4a\kappa}{4a\kappa}) \sigma_3 + \mathrm{i} (\cos 2a\kappa - \tfrac{\sin 2a\kappa}{2a\kappa}) \sigma_2 \Big). \tag{S9}$$

$$\begin{aligned} S_{\text{L}}^{(2,2)} = \psi \frac{\mathrm{i}\lambda^2}{8\pi^2 R^2 \kappa^3} \Big(&\cos 4a\kappa + 2a\kappa \sin 4a\kappa - 4a^2\kappa^2 - 1 \\ &+ \mathrm{i} 4a\kappa (\sin 2a\kappa - a\kappa \cos 2a\kappa) \sigma_2 \Big). \end{aligned} \tag{S10}$$

S_{R} up to λ^2.

$$S_{\text{R}}^{(1,1)} = S_{\text{L}}^{(1,1)}, \qquad S_{\text{R}}^{(2,1)} = \left(S_{\text{L}}^{(2,1)} \right)^\dagger, \qquad S_{\text{R}}^{(2,2)} = S_{\text{L}}^{(2,2)}. \tag{S11}$$

Analytical Results on $\mathbb{R}$.

$$V_{\text{ind}}^{\mathbb{R}} = -\frac{4\kappa\lambda \sin 2a\kappa}{4\kappa^2 + \lambda^2 \sin^2 2a\kappa} (\lambda \cos 2a\kappa + \lambda\sigma_1 + 2\kappa\sigma_2). \tag{S12}$$

$$\begin{aligned} S_{\text{L}}^{\mathbb{R}} = \psi \frac{4a\kappa\lambda}{4\kappa^2 + \lambda^2 \sin^2 2a\kappa} \Bigg(&2\kappa - \mathrm{i}\lambda \left(1 - \frac{\sin 4a\kappa}{4a\kappa} \right) \sigma_3 \\ &+ \frac{\sin 2a\kappa}{a} \sigma_1 + \lambda \left(\cos 2a\kappa - \frac{\sin 2a\kappa}{2a\kappa} \right) \sigma_2 \Bigg). \end{aligned} \tag{S13}$$

$$S_{\text{R}}^{\mathbb{R}} = \left(S_{\text{L}}^{\mathbb{R}} \right)^\dagger. \tag{S14}$$