Supplementary Material to the article "Effect of dephasing on the current through a periodically driven quantum point contact"

COUPLED GKSL EQUATIONS FOR ONSAGER STRINGS

The Hamiltonian (3) is a time-dependent linear combinations of operators $\sigma_j^x\sigma_{j+1}^x$ and $\sigma_j^y\sigma_{j+1}^y$. The commutation relations of these operators with Onsager strings (8) read

$$
\begin{aligned}
&[\sigma_j^x\sigma_{j+1}^x, A_i^n] = -4(B_{i+1}^{1-n}\delta_{ij} + B_i^{n-1}\delta_{i,j-n+1}), && [\sigma_j^y\sigma_{j+1}^y, A_i^n] = -4(B_{i-1}^{-1-n}\delta_{i,j+1} + B_i^{n+1}\delta_{i,j-n}),\\
&[\sigma_j^x\sigma_{j+1}^x, A_i^{-n}] = 4(B_{i-1}^{n+1}\delta_{i,j+1} + B_i^{-n-1}\delta_{i,j-n}), && [\sigma_j^y\sigma_{j+1}^y, A_i^{-n}] = 4(B_i^{1-n}\delta_{i,j-n+1} + B_{i+1}^{n-1}\delta_{i,j}),\\
&[\sigma_j^x\sigma_{j+1}^x, B_i^n] = A_{i+1}^{1-n}\delta_{i,j} - A_i^{n+1}\delta_{i,j-n}, && [\sigma_j^y\sigma_{j+1}^y, B_i^n] = A_{i-1}^{-n-1}\delta_{i,j+1} - A_{i+1}^{n-1}\delta_{i,j-n+1},\\
&[\sigma_j^x\sigma_{j+1}^x, B_i^{-n}] = A_i^{1-n}\delta_{i,j-n+1} - A_{i-1}^{n+1}\delta_{i,j+1}, && [\sigma_j^y\sigma_{j+1}^y, B_i^{-n}] = A_i^{-1-n}\delta_{i,j-n} - A_{i+1}^{n-1}\delta_{i,j},\\
&[\sigma_j^x\sigma_{j+1}^x, A_i^0] = 4(B_{i-1}^1\delta_{i,j+1} + B_i^{-1}\delta_{i,j}), && [\sigma_j^y\sigma_{j+1}^y, A_i^0] = -4(B_{i-1}^{-1}\delta_{i,j+1} + B_i^1\delta_{i,j}),
\end{aligned}
\tag{S1}
$$

where $1 \le n \le 2L-1$, $1 \le j \le 2L-n$ and we adopt the convention $B_j^0 = A_j^{\pm 2L} = B_j^{\pm 2L} = 0$.

Further, the action of the dissipation superoperator (2), (6) on Onsager strings read

$$
\mathcal{D}A_j^0 = 0, \qquad \mathcal{D}A_j^{\pm n} = -4\gamma A_j^{\pm n}, \qquad \mathcal{D}B_j^{\pm n} = -4\gamma B_j^{\pm n}, \qquad n \ge 1. \tag{S2}
$$

Plugging eqs. (S1), (3), (4), (5) and (S2) into the GKSL equation (1), one obtains an explicit form of coupled GKSL equations for Onsager strings.

ZZ-DEPHASING

In Fig. S1 we present results for a different set of Lindblad operators, namely

$$
l_j = \sigma_j^z\sigma_{j+1}^z, \qquad j = 1, 2, \dots, 2L-1. \tag{S3}
$$

One can see that there is no qualitative difference with the case of Lindblad operators (6) considered in the main text.

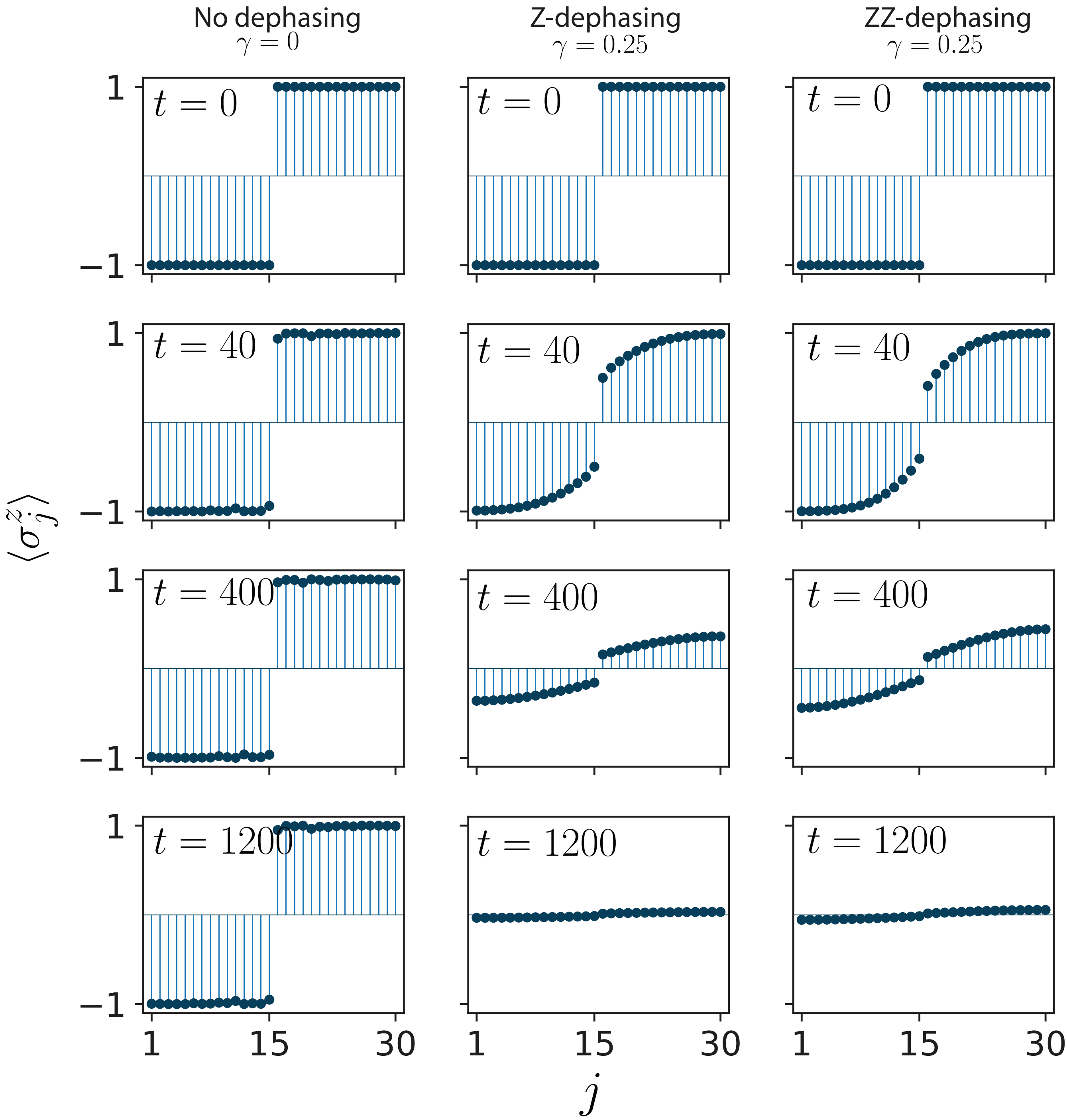

Figure S1: Snapshots of the magnetization profile: in the absence of dephasing (left column), in the presence of Z-dephasing described by eq. (6) (middle column), in the presence of ZZ-dephasing described by eq. (S3) (right column). The total number of spins is $2L = 30$, the driving frequency is $\omega = 2.5$.