

# Supplementary Material to the article “Dynamics of “breathing” skyrmions”

## 1 Parameters and structure of a magnetic skyrmion

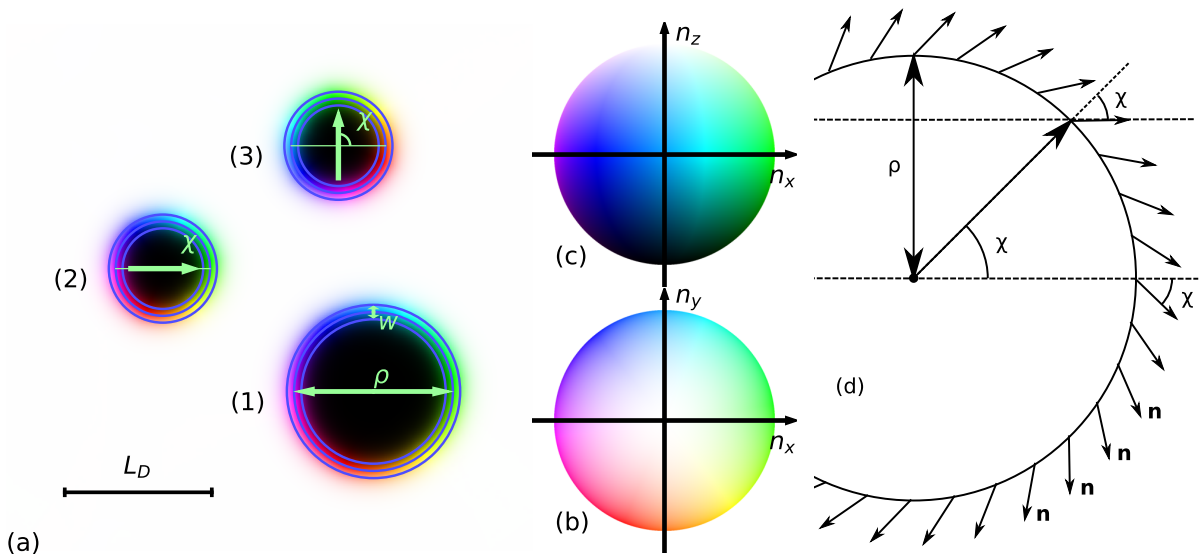


Fig. S1: (a) - equilibrium skyrmion in the absence of an external magnetic field (1) and in the field  $B = 0.032B_0$  (2). On the image of the skyrmion (1) its radius  $\rho$  and domain wall thickness  $w$  are shown. The scale is specified by a segment of length  $L_D$ . In (1) and (2) there are Neél-type skyrmions with helicity  $\chi = 0$ . For comparison, a Bloch-type skyrmion (3) with  $\chi = \pi/2$  is depicted, obtained for the Dzyaloshinskii–Moriya (DM) interaction density  $w_{DM} = D\mathbf{n} \cdot (\nabla \times \mathbf{n})$ . The direction of the magnetization vector  $\mathbf{n}$  is coded by color: the direction along (against) the  $z$  axis corresponds to white (black) color, in the  $xy$  plane it is coded as shown in two projections of the Bloch sphere in the insets (b) and (c). Inset (d) shows magnetization vectors lying in the  $xy$  plane for points distant from the center by the skyrmion radius  $\rho$ . Helicity  $\chi$  is defined as the angle between these vectors and the normal to the domain wall. Obviously,  $\chi$  is equal to the angle between the direction of the  $x$  axis and the radius vector of the point at which the magnetization is codirectional to the  $x$  axis. This radius vector and helicity are marked in the images of skyrmions (2) and (3) in the inset (a).

## 2 Assessment of calculation accuracy and choice of finite element size

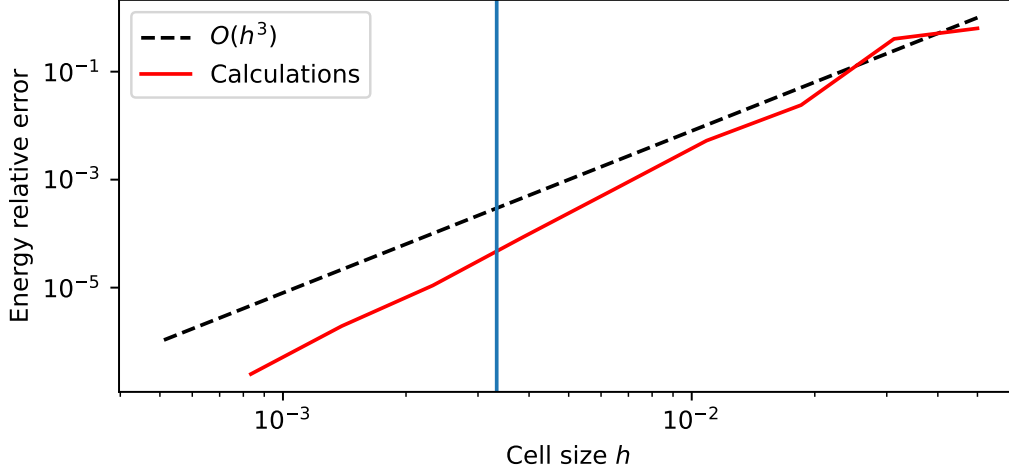


Fig. S2 :Relative error in micromagnetic energy calculation as a function of the length of the side  $h$  of the square, playing the role of a finite element. The black and red lines show the asymptotic behavior of  $O(h^3)$  and the result of the numerical calculation, respectively. The calculation was carried out for the parameters from the main text (with the magnetic field turned on). An ansatz with equilibrium parameters was used for magnetization. Since the analytical value of the energy is not known, as an estimate of the exact value we used energy for the smallest element size. The blue vertical line shows the element size used to model dynamics in the main text.

## 3 Skyrmion relaxation taking into account the possibility of changing the radius, domain wall thickness and helicity

Equations of skyrmion dynamics taking into account the change radius  $\rho$ , domain wall thickness  $w$  and helicity  $\chi$ :

$$\begin{cases} \alpha \frac{2\pi I_3}{w^2} \dot{\rho} + \alpha \frac{2\pi^3}{6} \dot{w} & + \frac{2\pi I_3}{w} \dot{\chi} = \frac{\partial E}{\partial \rho}, \\ \alpha \frac{2\pi^3}{6} \dot{\rho} + \alpha \frac{2\pi^3 \rho}{6w} \dot{w} & + \frac{2\pi^3 w}{6} \dot{\chi} = \frac{\partial E}{\partial w}, \\ -\frac{2\pi I_3}{w} \dot{\rho} - \frac{2\pi^3 w}{6} \dot{w} & + \alpha 2\pi I_3 \dot{\chi} = \frac{\partial E}{\partial \chi}. \end{cases}$$

The results of calculating the relaxation of the skyrmion when the magnetic field is turned on and off are presented in the figures S3 and S4 respectively.

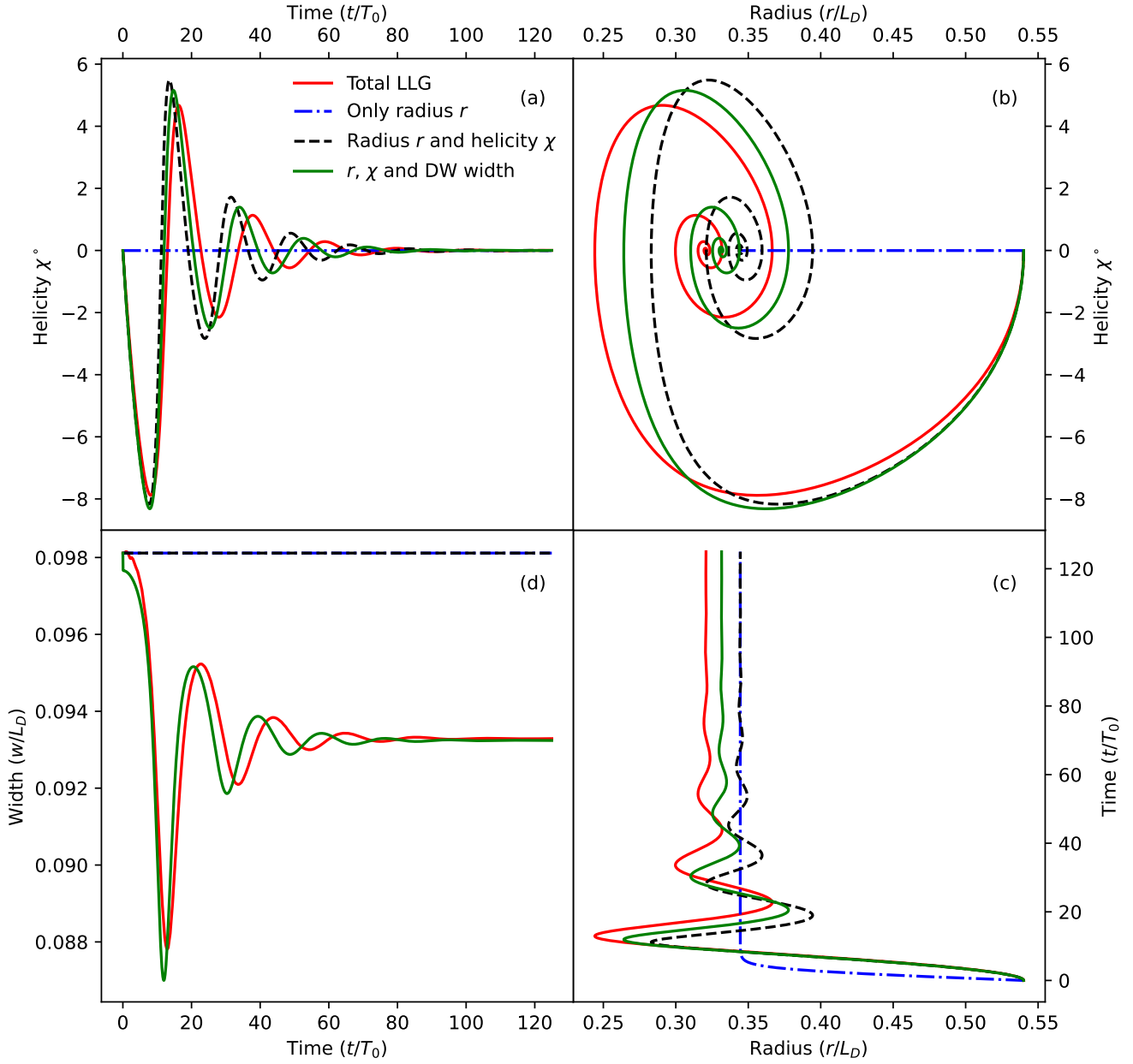


Fig. S3: (Color online) Skymion relaxation when an external magnetic field  $B = 0.032B_0$  is turned on (a) is the helicity, (c) is the radius, (d) is the domain wall width as a function of time. (b) is the helicity-radius phase diagram. The initial state corresponds to an equilibrium skyrmion in the absence of a magnetic field. Red solid, green solid, black dotted and blue dash-dotted lines are solutions obtained using the full system of Landau–Lifshitz–Gilbert (LLG) equations, the generalized Thiel equation, taking into account the possibility of changing helicity, domain wall width and radius, helicity and radius, and radius only, respectively.

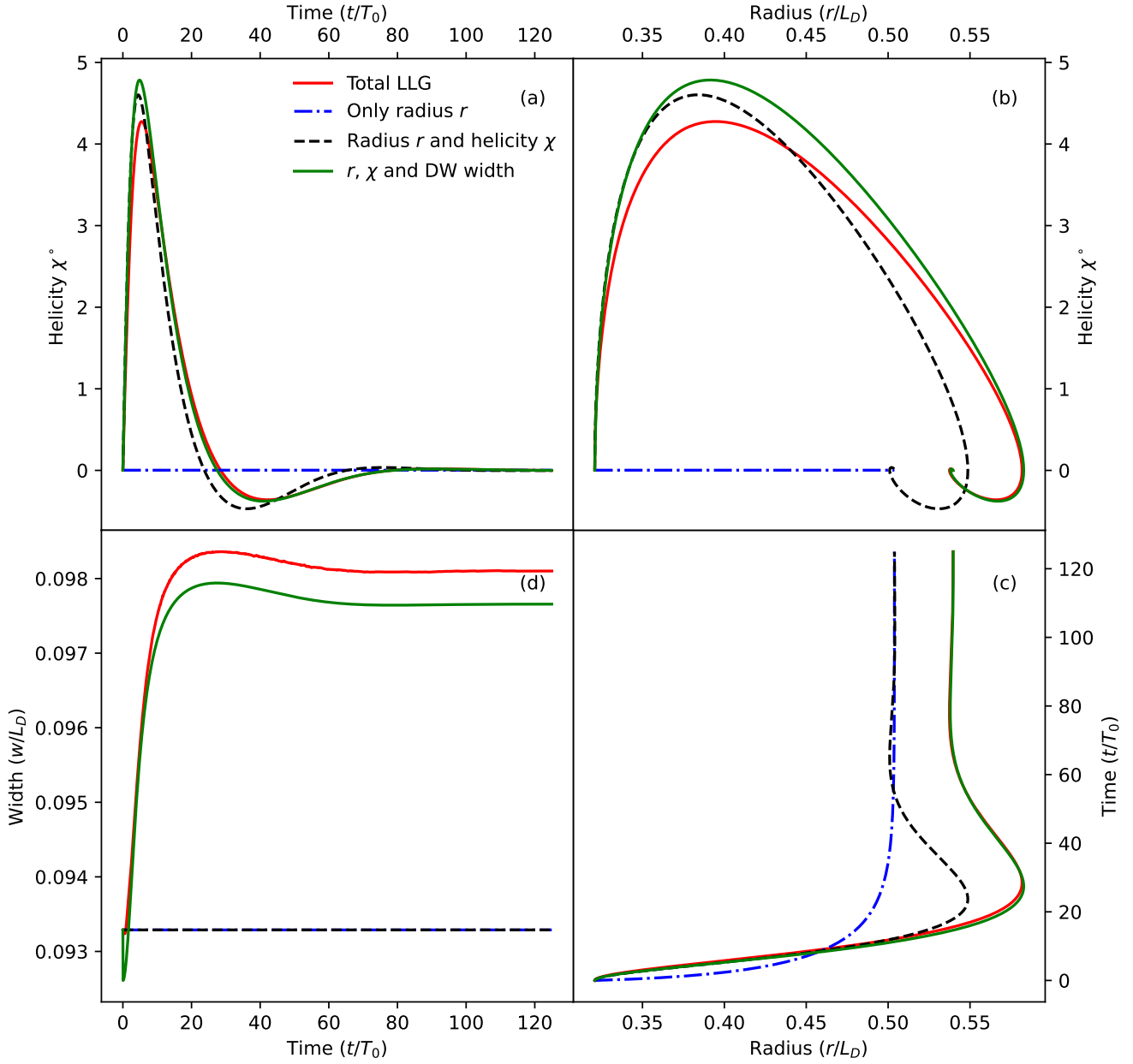


Fig. S4: Relaxation of a skyrmion, which was in an equilibrium state in an external magnetic field  $B = 0.032B_0$ , when the field is turned off. (a) is the helicity, (c) is the radius, (d) is the domain wall width as a functions of time. (b) is the helicity-radius phase diagram. The initial state corresponds to an equilibrium skyrmion in the absence of a magnetic field. Red solid, green solid, black dotted and blue dash-dotted lines are solutions obtained using the full system of LLG equations, the generalized Thiel equation, taking into account the possibility of changing helicity, domain wall width and radius, helicity and radius, and radius only, respectively.