Supplementary Material to the article
"Plasma response of a metallic “lattice” metasurface on a substrate"

I. TABLE S1. PARAMETERS OF THE SAMPLES USED IN THE EXPERIMENT

№	d, μm	a, μm	w, μm
1	103	100	10
2	103	150	10
3	103	200	10
4	103	300	10
5	103	300	5
6	103	300	30
7	213	200	10
8	213	100	10

II. DERIVATION OF THE TRANSMISSION FORMULA FOR AN EFFICIENT TWO-DIMENSIONAL SYSTEM

The electrons of a two-dimensional gas are considered in the Drude model, with the only exception that m_{eff} refers to the effective mass, the other variables are used in the usual sense: n_{eff} — effective two-dimensional electron concentration, τ — relaxation time, σ and μ are given by classical formulas:

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}; \qquad \sigma_0 = n_{\mathrm{eff}} e \mu; \qquad \mu = e\frac{\tau}{m_{\mathrm{eff}}}; \tag{S1}$$

Now, to find out how the field is distributed, consider an electromagnetic wave falling from a vacuum onto a plate with a frequency of ω:

$$E = E_x \exp(-i(\omega t - k_z z)); \tag{S2}$$

Part of the wave will be reflected, part will pass through the plate and another part will be absorbed into the DES. From Maxwell's equations for a homogeneous medium, we can describe the field distribution in the following form (S3), where $k_z = \frac{\omega}{c}$ and $k_{1z} = \frac{\sqrt{\varepsilon}\omega}{c}$.

$$E(z) = \begin{cases} \exp(ik_z z) + r\exp(-ik_z z) & z > 0 \\ a_1 \exp(ik_{1z} z) + a_2 \exp(-ik_{1z} z) & -d < z < 0 \\ t\exp(ik_z z) & z < -d \end{cases} \tag{S3}$$

Now let's take into account the boundary conditions, for $z = 0$ and $z = -d$. Using Ohm's law: $j = \sigma E$, and considering σ isotropic, we get the system:

$$\begin{cases} E\,(+0) = E\,(-0) \\ E\,(-d+0) = E\,(-d-0) \\ \frac{\partial E}{\partial x}\Big|_{z=-0}^{z=+0} = -i\frac{4\pi\omega}{c^2}\sigma E \\ \frac{\partial E}{\partial x}\Big|_{z=d+0}^{z=d-0} = 0 \end{cases} \tag{S4}$$

From the system, we find the coefficient t, and since $T = t \cdot t^*$, here * means complex conjugation, we finally get:

$$T = \left| \frac{2}{(1 + \frac{\sigma}{\varepsilon_0 c})(\cos\frac{\sqrt{\varepsilon}\omega}{c}d - \frac{i}{\sqrt{\varepsilon}}\sin\frac{\sqrt{\varepsilon}\omega}{c}d) + (\cos\frac{\sqrt{\varepsilon}\omega}{c}d - i\sqrt{\varepsilon}\sin\frac{\sqrt{\varepsilon}\omega}{c}d)} \right|^2, \tag{S5}$$

In the plasma limit of $\omega\tau \gg 1$, the conductivity turns out to be purely imaginary and is equal to:

$$\sigma(\omega) = i\frac{ne^2}{m\omega} \tag{S6}$$

In the absence of hybridization of plasma resonance with substrate resonances ($\omega_p \ll \pi/\sqrt{\varepsilon}d$), the transmission spectrum has resonance at the plasma frequency:

$$\omega_{\mathrm{2D}}^2 = \frac{ne^2}{m\varepsilon_0\left(\varepsilon - 1\right)d}. \tag{S7}$$

For a metal surface based on a square lattice with a period of a and a band width of w, the plasma frequency is given by the formula:

$$\omega_p^2 = \frac{2\pi c^2}{ad\left(\varepsilon - 1\right)\ln(a/w)}, \qquad \omega_p \ll \frac{\pi c}{\sqrt{\varepsilon}d}. \tag{S8}$$

Using effective mass and concentration formulas:

$$m_{\mathrm{eff}} = \frac{\mu_0 w e^2 n_s}{2\pi}\ln\frac{a}{w}, \qquad n_{\mathrm{eff}} = n_s w/a; \tag{S9}$$

We will get an expression for the effective conductivity:

$$\sigma_{eff} = i\frac{2\pi}{\omega\mu_0 a\ln\left(\frac{a}{w}\right)} \tag{S10}$$

It is important to note that neither the plasma frequency ($S8$) nor the conductivity ($S10$) depend on the thickness of the deposited metal and the concentration of electrons in it. Substituting the resulting conductivity into the formula ($S5$) we will get the transmission spectrum of an effective two-dimensional system. The results are shown in Fig. (S1). Here the dots show experimental data, solid curves — theoretical spectra described by the formula (S5).

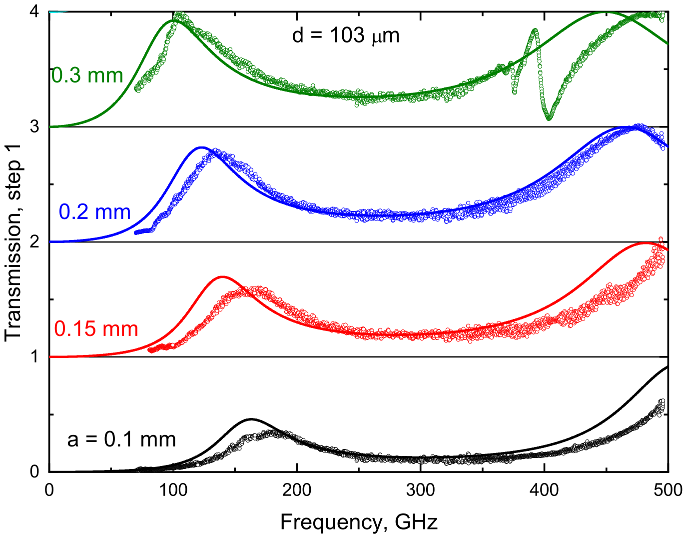

FIG. S1. Transmission spectra for samples with a silicon substrate thickness of $d = 103$ microns and a strip width of $w = 10$ microns for grid periods of $a = 100,\ 150,\ 200,\ 300$ microns. Solid lines show the transmission functions described by the formula (S5).

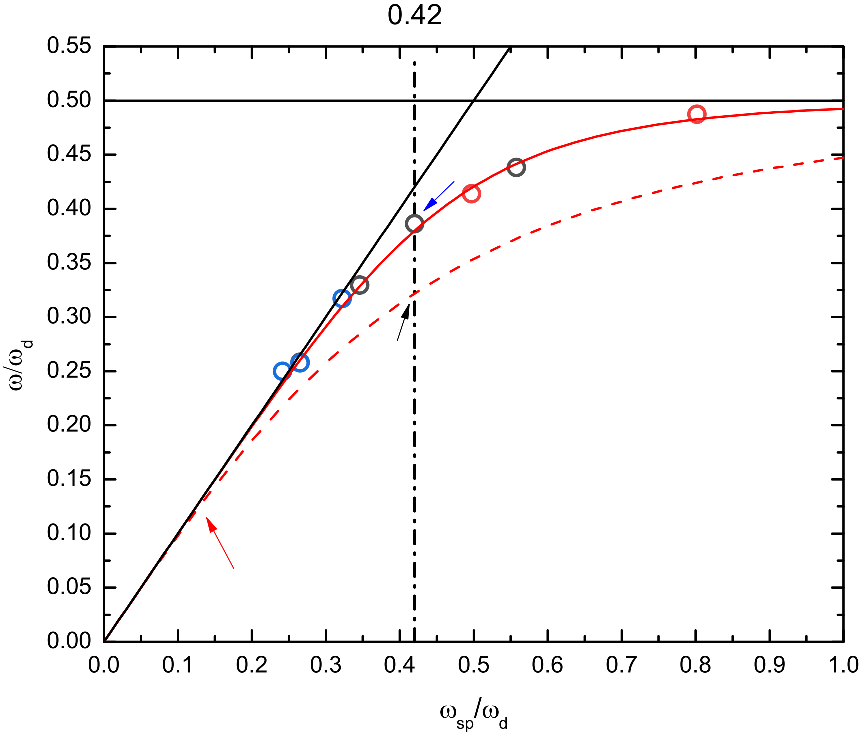

FIG. S2. A dimensionless graph, where the ratio of the plasma frequency to the frequency of the first Fabry-Perot resonance is plotted along the abscissa axis, ω_p/ω_1, and along the ordinate axis — the ratio of the resonance frequency to the frequency of the first Fabry-Perot resonance, ω/ω_1. Dots of different colors show the results of experiments conducted on samples with different geometries of the metal lattice. The red dotted line shows the theoretical dependence $\omega = \frac{\omega_p}{\sqrt{1+A^2}}$, solid— experimentally established dependence $\omega = \frac{\omega_p}{\sqrt[4]{1+A^4}}$.

It can be seen that the positions of the resonances of the metasurface do not agree well with the theoretical description for an effective electronic system. To explain this effect, consider a dimensionless graph (S2). The dots on it show experimental data for different samples. The black inclined line corresponds to the plasma frequency formula, and the black horizontal line corresponds to the frequency of the first minimum transmission of an empty substrate.

As can be seen, there is a hybridization between the plasma resonance and the resonance of the substrate. It turned out that the experimental results are well described by the following formula:

$$\omega = \frac{\omega_p}{\sqrt[4]{1+A^4}}, \tag{S11}$$

I must say right away that we do not have an analytical theory explaining this dependence. For a real two-dimensional system, hybridization is described by another formula:

$$\omega = \frac{\omega_p}{\sqrt{1+A^2}} \tag{S12}$$

It is shown on the graph with a dotted red line. It can be seen from the data obtained that the mechanism of hybridization of plasma resonance with substrate resonances in the case of a metal grid differs significantly from the case of a door electronic system. For clarity, in Figure (S2), a vertical line is drawn corresponding to the same conductivity and plasma frequency for the metal grid and DES. This line passes through the experimental point for a sample with a period of $a = 150$ µm and a band width of $w = 10$ µm. It can be seen that at the same plasma frequency, the resonance of our system will be at a higher frequency than the resonance of a similar effective two-dimensional system. What is observed in Fig. (S1) It is precisely because of the different hybridization mechanisms that a difference appears in the position of the maxima in the transmission spectra in Fig. (S1).

From the graph (S2) it can be seen at what values of the plasma frequency the difference in the position of the transmission resonances for the metal grid and DES should disappear. This moment is shown by the red arrow in Fig. (S2). It corresponds to the absence of hybridization of plasma resonance in DES and Fabry-Perot resonances of the substrate. Unfortunately, for our structures on silicon substrates with a thickness of 103 microns, this mode occurs at frequencies below 50 GHz. This frequency range does not allow the use of open quasi-optical circuits.

III. NUMERICAL MODELING OF ONE-DIMENSIONAL LATTICES

Transmission spectra were modeled for samples on the surface of which a one-dimensional lattice was used instead of a two-dimensional grid. In Fig.(S3) it can be seen that in the case of polarization coinciding with the direction of the bands, the one-dimensional lattice behaves like a two-dimensional one, and when falling across the bands, it simply behaves like a plate of pure silicon (S4). Qualitatively, this can be explained by the fact that metal strips simulate the kinetic inductance of a two-dimensional electronic system. And the bands perpendicular to the incident polarization do not make a noticeable contribution to the transmission spectrum.

Thus, this effect can be similarly observed in the case of a one-dimensional lattice, taking into account the control of incident polarization, and a two-dimensional lattice simply removes the dependence on polarization, making measurements more convenient.

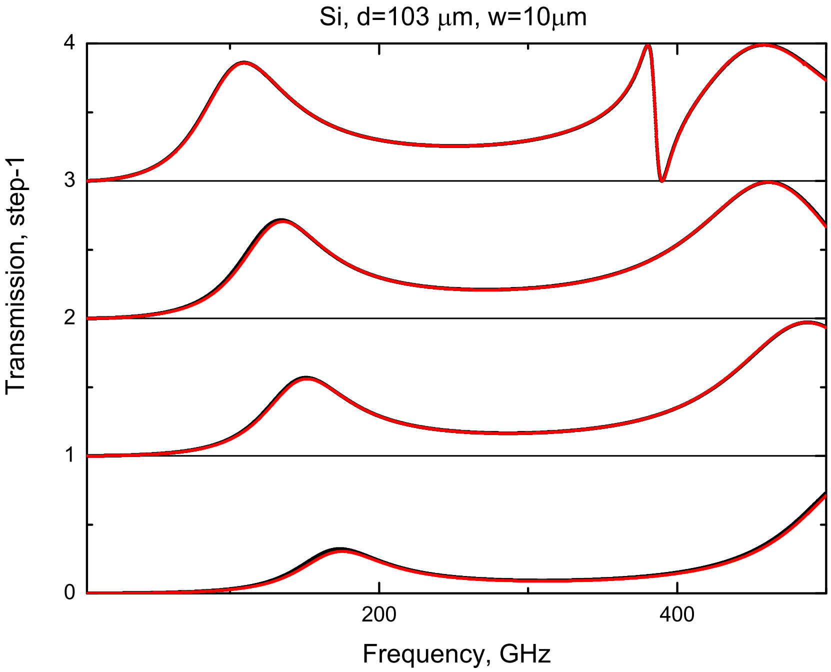

FIG. S3. Transmission spectra for silicon-based metasurfaces. The transmission spectra for a two-dimensional golden grid with periods are shown in red: 300, 200, 150 and 100 microns. The transmission spectra for one-dimensional gold grids are shown in black, and similar periods for polarization directed along the grid bands.

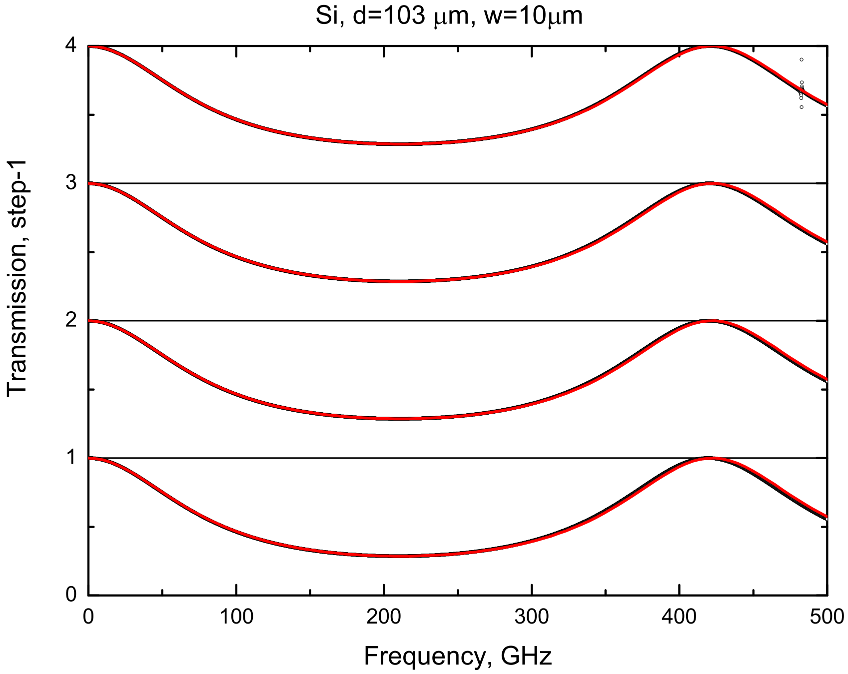

FIG. S4. Transmission spectra for silicon-based metasurfaces. The transmission spectra for the silicon wafer are shown in red according to the Fabry-Perot transmission formula. Black transmission spectra of a one-dimensional golden grid with periods: 300, 200, 150 and 100 microns in polarization perpendicular to the grid bands.