Supplementary Material to the article "Perturbations in Horndeski theory above anisotropic cosmological background"

Appendix A

In this Appendix we collect the expressions for coefficients A_i entering the quadratic action. Here and below the small Latin indices run the values a, b, c and we consider that $i \neq j \neq k$:

$$A_1 = -6G_4 + 12G_{4X}((\dot{\pi}))^2, \tag{1}$$

$$A_2 = 2G_4, \tag{2}$$

$$\begin{aligned} A_3 = {} & F_X((\dot{\pi}))^2 + 2F_{XX}((\dot{\pi}))^4 + 4\left(H_a + H_b + H_c\right) K_X((\dot{\pi}))^3 - K_\pi((\dot{\pi}))^2 \\ & - K_{X\pi}((\dot{\pi}))^4 + 2\left(H_a + H_b + H_c\right) K_{XX}((\dot{\pi}))^5 - 2\left(H_aH_b + H_aH_c + H_bH_c\right) G_4 \\ & + 14\left(H_aH_b + H_aH_c + H_bH_c\right) G_{4X}((\dot{\pi}))^2 + 32\left(H_aH_b + H_aH_c + H_bH_c\right) G_{4XX}((\dot{\pi}))^4 \\ & - 10\left(H_a + H_b + H_c\right) G_{4X\pi}((\dot{\pi}))^3 - 2\left(H_a + H_b + H_c\right) G_{4\pi}\left(\dot{\pi}\right) \\ & + 8\left(H_aH_b + H_aH_c + H_bH_c\right) G_{4XXX}((\dot{\pi}))^6 - 4\left(H_a + H_b + H_c\right) G_{4XX\pi}((\dot{\pi}))^5, \end{aligned} \tag{3}$$

$$\begin{aligned} A_4^i = {} & 2K_X((\dot{\pi}))^3 - 2\left(H_j + H_k\right) G_4 + 8\left(H_j + H_k\right) G_{4X}((\dot{\pi}))^2 \\ & - 4G_{4X\pi}((\dot{\pi}))^3 - 2G_{4\pi}\left(\dot{\pi}\right) + 8\left(H_j + H_k\right) G_{4XX}((\dot{\pi}))^4, \end{aligned} \tag{4}$$

$$A_5 = -\frac{1}{3}A_1 \tag{5}$$

$$A_6^i = -A_4^i \tag{6}$$

$$A_7 = \frac{1}{3}A_1 \tag{7}$$

$$\begin{aligned} A_8^i = {} & 2K_X((\dot{\pi}))^2 - 2G_{4\pi} + 4\left(H_j + H_k\right) G_{4X}\left(\dot{\pi}\right) + 8\left(H_j + H_k\right) G_{4XX}((\dot{\pi}))^3 \\ & - 4G_{4X\pi}((\dot{\pi}))^2, \end{aligned} \tag{8}$$

$$A_9^i = -A_8^i \tag{9}$$

$$A_{10}^i = -A_8^i \tag{10}$$

$$\begin{aligned} A_{11} = {} & -2F_X\left(\dot{\pi}\right) - 4F_{XX}((\dot{\pi}))^3 + 2K_{X\pi}((\dot{\pi}))^3 + 2K_\pi\left(\dot{\pi}\right) - 4\left(H_a + H_b + H_c\right) K_{XX}((\dot{\pi}))^4 \\ & - 6\left(H_a + H_b + H_c\right) K_X((\dot{\pi}))^2 - 12\left(H_bH_c + H_aH_c + H_aH_b\right) G_{4X}\left(\dot{\pi}\right) \\ & - 48\left(H_bH_c + H_aH_c + H_aH_b\right) G_{4XX}((\dot{\pi}))^3 + 16\left(H_a + H_b + H_c\right) G_{4X\pi}((\dot{\pi}))^2 \\ & + 2\left(H_a + H_b + H_c\right) G_{4\pi} - 16\left(H_aH_b + H_aH_c + H_bH_c\right) G_{4XXX}((\dot{\pi}))^5 \\ & + 8\left(H_a + H_b + H_c\right) G_{4XX\pi}((\dot{\pi}))^4, \end{aligned} \tag{11}$$

$$
\begin{aligned}
A_{12} &= 2F_X\,(\dot\pi) + 2\,(H_a+H_b+H_c)\,K_X((\dot\pi))^2 - 2K_\pi\,(\dot\pi)\\
&+ 4\,(H_bH_c+H_aH_c+H_aH_b)\,G_{4X}\,(\dot\pi) + 2G_{4\pi\pi}\,(\dot\pi) - 2G_{4\pi}H_a\\
&+ 8\,(H_aH_b+H_aH_c+H_bH_c)\,G_{4XX}((\dot\pi))^3 - 8\,(H_a+H_b+H_c)\,G_{4X\pi}((\dot\pi))^2,
\end{aligned}
\tag{12}
$$

$$
A^{ij}_{13} = -2G_{4\pi} + 4G_{4X}\,(\ddot\pi) + 4G_{4X}H_k\,(\dot\pi) + 4G_{4X\pi}((\dot\pi))^2 + 8G_{4XX}\,(\ddot\pi)\,((\dot\pi))^2
\tag{13}
$$

$$
\begin{aligned}
A_{14} &= 2F_{XX}((\dot\pi))^2 + F_X - K_\pi - K_{X\pi}((\dot\pi))^2 + 2\,(H_a+H_b+H_c)\,K_{XX}((\dot\pi))^3\\
&+ 2\,(H_a+H_b+H_c)\,K_X\,(\dot\pi) - 6\,(H_a+H_b+H_c)\,G_{4X\pi}\,(\dot\pi)\\
&+ 16\,(H_aH_b+H_aH_c+H_bH_c)\,G_{4XX}((\dot\pi))^2 + 2\,(H_aH_b+H_aH_c+H_bH_c)\,G_{4X},\\
&- 4\,(H_a+H_b+H_c)\,G_{4XX\pi}((\dot\pi))^3 + 8\,(H_aH_b+H_aH_c+H_bH_c)\,G_{4XXX}((\dot\pi))^4,
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
A^{i}_{15} &= -F_X + K_\pi - K_{X\pi}((\dot\pi))^2 - 2K_{XX}\,(\ddot\pi)\,((\dot\pi))^2 - 2K_X\,(\ddot\pi)\\
&- 2\,(H_j+H_k)\,K_X\,(\dot\pi) + 6G_{4X\pi}\,(\ddot\pi) + 6\,(H_j+H_k)\,G_{4X\pi}\,(\dot\pi)\\
&+ 4G_{4XX\pi}\,(\ddot\pi)\,((\dot\pi))^2 - 4\,(H_j+H_k)\,G_{4XX\pi}((\dot\pi))^3 - 8\,(H_j+H_k)\,G_{4XXX}\,(\ddot\pi)\,((\dot\pi))^3\\
&- 12\,(H_j+H_k)\,G_{4XX}\,(\dot\pi)\,(\ddot\pi) - 4\left(H_j{}^2 + \left(\dot H_j\right) + H_k{}^2 + \left(\dot H_k\right) + 3H_jH_k\right)G_{4XX}((\dot\pi))^2\\
&+ 2G_{4X\pi\pi}((\dot\pi))^2 - 2\left(\left(\dot H_j\right) + H_j{}^2 + \left(\dot H_k\right) + H_k{}^2 + H_jH_k\right)G_{4X},
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
A_{17} &= F_\pi - 2F_{X\pi}((\dot\pi))^2 - 8\,(H_bH_c+H_aH_c+H_aH_b)\,G_{4X\pi}((\dot\pi))^2\\
&+ 2\,(H_a+H_b+H_c)\,G_{4\pi\pi}\,(\dot\pi) - 8\,(H_aH_b+H_aH_c+H_bH_c)\,G_{4XX\pi}((\dot\pi))^4\\
&+ 4\,(H_a+H_b+H_c)\,G_{4X\pi\pi}((\dot\pi))^3 + K_{\pi\pi}((\dot\pi))^2 - 2\,(H_a+H_b+H_c)\,K_{X\pi}((\dot\pi))^3\\
&+ 2\,(H_aH_c+H_bH_c+H_aH_b)\,G_{4\pi},
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
A^{i}_{18} &= -2F_X\,(\dot\pi) - 2K_{X\pi}((\dot\pi))^3 + 2K_\pi\,(\dot\pi) - 4K_{XX}\,(\ddot\pi)\,((\dot\pi))^3 - 4K_X\,(\dot\pi)\,(\ddot\pi)\\
&- 4\,(H_a+H_b+H_c)\,K_X((\dot\pi))^2 - 4\left(\left(\dot H_j\right) + \left(\dot H_k\right) + (H_j+H_k)^2 + \right.\\
&\left. 2H_bH_c + 2H_aH_b + 2H_aH_c\right)G_{4X}\,(\dot\pi) + 4G_{4X\pi\pi}((\dot\pi))^3 + 8G_{4XX\pi}\,(\ddot\pi)\,((\dot\pi))^3\\
&+ 12G_{4X\pi}\,(\dot\pi)\,(\ddot\pi) + 8\,(H_a+H_b+H_c)\,G_{4X\pi}((\dot\pi))^2 + 2\,(H_j+H_k+2H_i)\,G_{4\pi}\\
&- 8\left((H_j+H_k)^2 + \left(\dot H_j\right) + \left(\dot H_k\right) + 2H_iH_j + 2H_iH_k + H_jH_k\right)G_{4XX}((\dot\pi))^3\\
&- 8\,(H_j+H_k)\,G_{4XX\pi}((\dot\pi))^4 - 16\,(H_j+H_k)\,G_{4XXX}\,(\ddot\pi)\,((\dot\pi))^4\\
&- 32\,(H_j+H_k)\,G_{4XX}\,(\ddot\pi)\,((\dot\pi))^2 - 4\,(H_j+H_k)\,G_{4X}\,(\ddot\pi)\,,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
A_{20} &= \frac{1}{2}F_{\pi\pi} - F_{X\pi\pi}((\dot{\pi}))^2 - 2F_{XX\pi}(\ddot{\pi})((\dot{\pi}))^2 - F_{X\pi}(\ddot{\pi}) \\
&- (H_a + H_b + H_c) F_{X\pi}(\dot{\pi}) + K_{\pi\pi}(\ddot{\pi}) + (H_a + H_b + H_c) K_{\pi\pi}(\dot{\pi}) + K_{X\pi\pi}(\ddot{\pi})((\dot{\pi}))^2 \\
&- (H_a + H_b + H_c) K_{X\pi\pi}((\dot{\pi}))^3 - 2(H_a + H_b + H_c) K_{XX\pi}(\ddot{\pi})((\dot{\pi}))^3 \\
&- \left((H_a + H_b + H_c)^2 + \left(\dot{H}_a\right) + \left(\dot{H}_b\right) + \left(\dot{H}_c\right)\right) K_{X\pi}((\dot{\pi}))^2 \\
&- 2(H_a + H_b + H_c) K_{X\pi}(\dot{\pi})(\ddot{\pi}) + \frac{1}{2}K_{\pi\pi\pi}((\dot{\pi}))^2 - 2\left(\frac{d}{dt}[H_a H_b + H_a H_c + H_b H_c]\right. \\
&\left. + {H_a}^2(H_b + H_c) + H_b^2(H_a + H_c) + H_c^2(H_a + H_b) + 3H_a H_b H_c\right) G_{4X\pi}(\dot{\pi}) \\
&+ 6(H_a + H_b + H_c) G_{4X\pi\pi}(\dot{\pi})(\ddot{\pi}) + 2\left((H_a + H_b + H_c)^2 + \left(\dot{H}_a\right)\right. \\
&\left. + \left(\dot{H}_b\right) + \left(\dot{H}_c\right)\right) G_{4X\pi\pi}((\dot{\pi}))^2 + 4(H_a + H_b + H_c) G_{4XX\pi\pi}(\ddot{\pi})((\dot{\pi}))^3 \\
&- 4(H_a H_b + H_a H_c + H_b H_c) G_{4XX\pi\pi}((\dot{\pi}))^4 - 8(H_a H_b + H_a H_c + H_b H_c) G_{4XXX\pi}(\ddot{\pi})((\dot{\pi}))^4 \\
&- 16(H_a H_b + H_a H_c + H_b H_c) G_{4XX\pi}(\ddot{\pi})((\dot{\pi}))^2 - 4\left(\frac{d}{dt}[H_a H_b + H_a H_c + H_b H_c]\right. \\
&\left. + {H_a}^2(H_b + H_c) + H_b(H_a + H_c) + H_c^2(H_a + H_b) + 3H_a H_b H_c\right) G_{4XX\pi}((\dot{\pi}))^3 \\
&+ 2(H_a + H_b + H_c) G_{4X\pi\pi\pi}((\dot{\pi}))^3 - 2(H_a H_b + H_a H_c + H_b H_c) G_{4X\pi}(\ddot{\pi}) \\
&+ \left(\left(\dot{H}_c\right) + {H_c}^2 + \left(\dot{H}_b\right) + {H_b}^2 + H_b H_c + \left(\dot{H}_a\right) + {H_a}^2 + 2H_a H_c + H_a H_b\right) G_{4\pi\pi},
\end{aligned} \tag{18}
$$

$$
B^{ab} = \frac{1}{3}(H_b - H_a)A_1, \tag{19}
$$

$$
B^{bc} = \frac{1}{3}(H_c - H_b)A_1, \tag{20}
$$

$$
B^{ac} = \frac{1}{3}(H_c - H_a)A_1. \tag{21}
$$

Appendix B

$$
D_1 = \frac{2}{3}(H_a - H_b)\bar{H}_a\bar{H}_b + \frac{2}{3}(H_c - H_b)\bar{H}_b\bar{H}_c + \frac{2}{3}(H_a - H_c)\bar{H}_a\bar{H}_c + \tag{22}
$$

$$
+ \frac{1}{3}\sum_{\substack{i,j=a,b,c \\ i\neq j}} \frac{d}{dt}\left[\bar{H}_i\bar{H}_j\right], \tag{23}
$$

$$
D_2 = \frac{1}{abc}\frac{d}{dt}[abcD_1], \tag{24}
$$

$$
A = \frac{2}{3}A_1\left(\bar{H}_a\bar{H}_b + \bar{H}_a\bar{H}_c + \bar{H}_b\bar{H}_c\right) - \frac{9}{2}\frac{{A_4}^2}{A_3}. \tag{25}
$$

In this section we denote

$$
A_4 = \frac{1}{3}\sum_{l=a,b,c} A_4^l\bar{H}_l, \quad A_8 = \frac{1}{3}\sum_{l=a,b,c} A_8^l\bar{H}_l, \quad A_{18} = \sum_{l=a,b,c} A_{18}^l\bar{H}_l, \tag{26}
$$

and as usual $i \neq j \neq k$.

$$\Theta_i = \frac{1}{3} A_1 \left(\bar{H}_j + \bar{H}_k\right), \tag{27}$$

$$\Lambda_i = \frac{1}{3A_3} \left(9 A_4 A_8^i + A_1 A_{11} \left(\bar{H}_j + \bar{H}_k\right)\right), \tag{28}$$

$$\Xi_i = \frac{1}{2} \left[-2A_{13}^{ij} \bar{H}_j - 2A_{13}^{ik} \bar{H}_k - 4\eta A_2^{jk} \bar{H}_j \bar{H}_k - \eta \frac{d}{dt} \left[\frac{A_1 A_4 \left(\bar{H}_i + \bar{H}_k\right)}{A_3} \right] - \right.$$
$$\frac{1}{3} \frac{d}{dt} \left[\frac{A_1 A_{11} \left(\bar{H}_i + \bar{H}_k\right)}{A_3} \right] + \frac{1}{3A_3} \left(-3A_8^i + 2A_1 \eta \left(\bar{H}_j + \bar{H}_k\right)\right) \sum_{l=a,b,c} A_4^l \dot{\bar{H}}_l +$$
$$\left. + \frac{1}{3} \frac{A_1}{A_3} \left(A_{11} + 3\eta A_4\right) \left(\bar{H}_j + \bar{H}_k\right) H \left(\bar{H}_i - \bar{H}_j - \bar{H}_k\right) + \frac{1}{3A_3} A_1 A_{17} \left(\bar{H}_j + \bar{H}_k\right) \right], \tag{29}$$

$$\Pi_i = \frac{1}{2} \left[\frac{1}{2} \frac{d}{dt} \left[\frac{A_1 A_4 \left(\bar{H}_j + \bar{H}_k\right)}{A_3} \right] - \frac{1}{3} \frac{A_1}{A_3} \left(\bar{H}_j + \bar{H}_k\right) \sum_{l=a,b,c} A_4^l \dot{\bar{H}}_l + \right.$$
$$\left. + \frac{1}{2} \frac{A_1 A_4 H}{A_3} \left(\bar{H}_j + \bar{H}_k\right) \left(\bar{H}_j + \bar{H}_k - \bar{H}_i\right) + 2A_2^{jk} \bar{H}_j \bar{H}_k \right], \tag{30}$$

$$\Sigma_i = \frac{1}{3} A_1 \eta \left(\bar{H}_j + \bar{H}_k\right) - A_8^i, \tag{31}$$

$$M = -\frac{1}{2} \frac{1}{abc} \frac{d}{dt} \left[abc \left(-\frac{1}{2} \frac{A_{11} A_{17}}{A_3} + \eta \left(\dot{\eta}\right) A + \eta \left(-A_{18} + 2 \sum_{l=a,b,c} A_8^l \, \dot{\bar{H}}_l - 3\dot{A}_8 - \right.\right.\right.$$
$$\left.\left.\left. -\frac{3}{2} \frac{A_{17} A_4}{A_3} - 9 A_8 H + \frac{1}{2} \frac{A_{11}}{A_3} \sum_{l=a,b,c} A_4^l \dot{\bar{H}}_l \right)\right)\right] - \frac{1}{4} \frac{{A_{17}}^2}{A_3} + A_{20} + \frac{1}{2} A \left(\dot{\eta}\right)^2 +$$
$$+ \left(\dot{\eta}\right) \left(-A_{18} + 2 \sum_{l=a,b,c} A_8^l \, \dot{\bar{H}}_l - 3\dot{A}_8 - \frac{3}{2} \frac{A_{17} A_4}{A_3} - 9 A_8 H + \frac{1}{2} \frac{A_{11}}{A_3} \sum_{l=a,b,c} A_4^l \, \dot{\bar{H}}_l \right) +$$
$$+ \eta \left(-\frac{1}{2} \frac{A_{17}}{A_3} \sum_{l=a,b,c} A_4^l \, \dot{\bar{H}}_l + \frac{1}{2} \frac{1}{abc} \frac{d}{dt} \left[\frac{abc A_{11}}{A_3} \sum_{l=a,b,c} A_4^l \, \dot{\bar{H}}_l \right] \right) +$$
$$+ \eta^2 \left(\frac{1}{2} A_1 D_2 + \frac{1}{2} D_1 \left(\dot{A}_1\right) - \frac{1}{4} \frac{1}{A_3} \left(\sum_{l=a,b,c} A_4^l \, \dot{\bar{H}}_l \right)^2 + \frac{3}{4} \frac{1}{abc} \frac{d}{dt} \left[\frac{abc A_4}{A_3} \sum_{l=a,b,c} A_4^l \, \dot{\bar{H}}_l \right] \right), \tag{32}$$

$$m = \left(A_1 D_2 + D_1 \dot{A}_1\right) - \frac{1}{A_3} \left(\sum_{l=a,b,c} A_4^l \, \dot{\bar{H}}_l \right)^2 + \frac{1}{abc} \frac{d}{dt} \left[\frac{abc A_4}{A_3} \sum_{l=a,b,c} A_4^l \, \dot{\bar{H}}_l \right]. \tag{33}$$