Supplementary Material to the article
"Shot noise in helical edge states in presence of a static magnetic defect"

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We provide general formulas describing the dependence of the transmission coefficient and the noise intensity on the magnetic flux through a helical interferometer with a static magnetic defect. The formulas are valid for an arbitrary tunnel coupling strength and an arbitrary scattering amplitude at the defect. For comparison, the corresponding formulas for a conventional (non-helical) ballistic interferometer based on a single-channel spinless wire are also given.

I. HELICAL INTERFEROMETER

General formulas for the conductance and the noise intensity are obtained by averaging the expressions (2), (3), (4) and (5) in the main text over the energy, using expressions for the amplitude (10). It is convenient to write the resulting formulas, using the parameters λ, R_θ and X instead of the parameters t, θ and ϕ, according to the following definitions:

$$t = e^{-\lambda}, \quad r = \sqrt{1-e^{-\lambda}}, \quad 0 < \lambda < \infty, \tag{S1}$$

$$R_\theta = \sin^2\theta, \tag{S2}$$

$$X = \cos^2(2\pi\phi). \tag{S3}$$

The conductance is proportional to the transmission coefficient averaged over the spin and the energy:

$$\tilde{\mathcal{T}} = \mathrm{Tr}\langle\hat{\mathcal{T}}\rangle_\varepsilon/2. \tag{S4}$$

Introducing also the average

$$\tilde{\mathcal{T}}_2 = \mathrm{Tr}\langle\hat{\mathcal{T}}\hat{\mathcal{T}}\rangle_\varepsilon, \tag{S5}$$

one can write the noise power and the Fano factor in the following form:

$$\mathcal{S} = 2\tilde{\mathcal{T}} - \tilde{\mathcal{T}}_2, \tag{S6}$$

$$\mathcal{F} = 1 - \frac{\tilde{\mathcal{T}}_2}{2\tilde{\mathcal{T}}}, \tag{S7}$$

Direct calculation of energy averages in formulas (S4) and (S5) yields

$$\tilde{\mathcal{T}} = \frac{\tanh\lambda}{D}\sum_{n=0}^{2} A_n \cosh^n(2\lambda)\,, \tag{S8}$$

$$\tilde{\mathcal{T}}_2 = \frac{\tanh\lambda}{D^3\cosh^2\lambda}\sum_{n=0}^{7} \tilde{A}_n \cosh^n(2\lambda)\,, \tag{S9}$$

where

$$
\begin{aligned}
D &= \cosh^2(2\lambda) - (1-R_\theta)X\,,\\
A_0 &= -(1-R_\theta)X\,,\\
A_1 &= R_\theta/2\,,\\
A_2 &= 1 - R_\theta/2\,,\\
\tilde{A}_0 &= X^2(R_\theta-1)^2R_\theta\,,\\
\tilde{A}_1 &= \frac{1}{4}X(R_\theta-1)(2X(R_\theta-1)-3R_\theta)(2X(R_\theta-1)-R_\theta)\,,\\
\tilde{A}_2 &= -\frac{3}{2}X(R_\theta-1)R_\theta^2\,,\\
\tilde{A}_3 &= X^2(R_\theta+3)(R_\theta-1)^2 + \frac{1}{4}X(R_\theta-8)R_\theta(R_\theta-1) - \frac{R_\theta^2}{4}\,,\\
\tilde{A}_4 &= \frac{1}{2}R_\theta\left(2X\left(R_\theta^2+R_\theta-2\right)+R_\theta-2\right)\,,\\
\tilde{A}_5 &= \frac{1}{4}(R_\theta^2 - 2X(R_\theta-1)(R_\theta^2-6))\,,\\
\tilde{A}_6 &= -(R_\theta-2)R_\theta\,,\\
\tilde{A}_7 &= 1 + \frac{1}{2}(R_\theta-2)R_\theta\,.
\end{aligned}
\tag{S10}
$$

In the absence of a magnetic defect, $\theta = 0$, the Fano factor and the transmission coefficient are related by equation

$$\mathcal{F} = \frac{1}{2}(1-\tilde{\mathcal{T}}^2). \tag{S11}$$

II. CONVENTIONAL INTERFEROMETER

In a case of a conventional single-channel spinless interferometer, the interference effects arise even without impurities due to scattering at the contacts [1]. We assume that the point contacts are characterized by the tunneling parameter, γ, [1, 2]. The cases $\gamma = 0$ and $\gamma = \infty$ correspond to tunnel contacts of various types, and $\gamma = 1$ to a metallic contact, as illustrated in the Fig. S1. In order to compare with the helical interferometer, we consider the

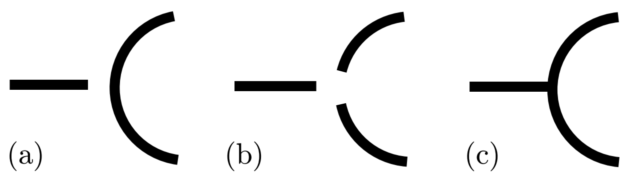

FIG. S1. Three limiting cases of contact to a conventional interferometer: (a) tunnel contact, $\gamma = 0$, (b) tunnel contact, $\gamma = \infty$, (c) metallic contact, $\gamma = 1$.

case of a *symmetric* interferometer with equal arm lengths. The energy-averaged transmission coefficient of such an interferometer [1] is given by

$$\tilde{\mathcal{T}}_{\text{conv}}(\phi,\gamma) = \frac{2\gamma\cos^2(\pi\phi)}{\gamma^2+\cos^2(\pi\phi)}\,. \tag{S12}$$

The Fano factor is calculated similarly to the case of the helical interferometer (detailed analysis, including the asymmetric case of non-equal arms, will be presented elsewhere):

$$\mathcal{F}_{\text{conv}}(\phi,\gamma) = 1 - \frac{\cos^2(\pi\phi)\left[1+11\gamma^2+2\gamma^4+(1+\gamma^2)\cos(2\pi\phi)\right]}{\left[1+2\gamma^2+\cos(2\pi\phi)\right]^2}. \tag{S13}$$

Several interesting properties of this expression are to be mentioned here. Firstly, $\mathcal{F}_{\text{conv}}(0,\gamma) = \mathcal{F}_{\text{conv}}(0,1/\gamma)$, which, in particular, means that the tunnel contacts in panels (a) and (b) of the Fig. S1 are indistinguishable in terms of noise and conductance for zero flux, $\phi = 0$. Secondly, there is a special point, $\phi = 1/2$, where, for a fixed γ, the conductance is exactly zero (see [3] and a more detailed discussion of this property in [1]), and the Fano factor to unity $\mathcal{T}_{1,\rm conv}(1/2,\gamma) = 0$, $\mathcal{F}_{\rm conv}(1/2,\gamma) = 1$. The behavior of the Fano factor around this point, for $|\delta\phi| = |\phi - 1/2| \ll 1$ and small γ is described by the expression

$$\mathcal{F}_{\rm conv}(1/2+\delta\phi,\gamma) \approx \frac{2-x^2+x^4}{2(1+x^2)^2}, \qquad \text{for} \quad \delta\phi \to 0, \gamma \to 0 \quad \text{and arbitrary} \quad x = \frac{\pi\delta\phi}{\gamma}. \tag{S14}$$

It means that the value of $\mathcal{F}_{\rm conv}$ depends on the order of taking the limits $\delta\phi \to 0$ and $\gamma \to 0$, and if we fix $\delta\phi$ and tend γ to zero, then instead of unity we get the usual expression for the tunnel contact $\mathcal{F}_{\rm conv} = 1/2$.

Away from the singular point $\phi = 1/2$, in three cases of contact shown in the Fig. S1 we have, respectively:

$$\begin{aligned} \mathcal{F}_{\rm conv}[\phi, 0] &= \frac{1}{2}\,, \\ \mathcal{F}_{\rm conv}[\phi, \infty] &= 1 - \frac{1}{2}\cos^2(\pi\phi)\,, \\ \mathcal{F}_{\rm conv}[\phi, 1] &= \frac{4\sin^2(\pi\phi)}{(\cos(2\pi\phi)+3)^2}\,. \end{aligned} \tag{S15}$$

At zero magnetic flux, $\phi = 0$, the Fano factor and transmission coefficient are related in the same way as in the helical case (see (S11)):

$$\mathcal{F}_{\rm conv} = \frac{1}{2}(1 - \tilde{\mathcal{T}}^2_{\rm conv})\,, \qquad \text{for} \quad \phi = 0. \tag{S16}$$

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[3] M. Büttiker, Y. Imry, and M. Y. Azbel, Phys. Rev. A **30**, 1982 (1984).