Supplementary Material to the article "Scattering of linear waves on a soliton"

A. Numerical scheme We solve numerically the nonlinear Schrödinger equation

$$i\partial_t\psi = -\frac{1}{2}\partial_x^2\psi + V(|\psi|^2)\psi\ , \tag{S1}$$

using 4-th order pseudo-spectral operator-splitting method [1, 2]. This method is unitary, stable, symplectic and therefore exceptionally accurate for computing small-valued high-order effects. The wave function at the next time step is given by

$$\psi(t+\Delta t) = \hat{\mathcal{U}}\,\psi(t)\ , \tag{S2}$$

where the evolution operator is replaced by discrete formula

$$\hat{\mathcal{U}} = \prod_{\alpha=1}^{4} \mathrm{e}^{-id_\alpha\Delta t\, V_\alpha(x)}\mathrm{e}^{-ic_\alpha\Delta t\,\hat{p}^2/2} + O(\Delta t^5). \tag{S3}$$

Here the product is ordered right-to-left, its parameters c_α and d_α are given in [1, 2], and the "potentials" V_β are computed using the "current" field $\psi(t)$ (multiplied by all operators with $\alpha \leq \beta$), i.e. $V_\beta = V(|\psi|^2)$. Eq. (S3) breaks the time interval Δt into c- and d-sub-intervals, on which the "kinetic" part and the "potential" part of the evolution operator are used respectively.

The numerical application of the formula (S3) is as follows. We introduce uniform lattice with N sites in the finite box of size L. The field values $\psi(t, x_j)$ are stored on the lattices sites $x_j = -L/2 + j\Delta x$, $0 \leq j < N$, where $\Delta x = L/N$. Typically, we use $L = 12000$ with $N = 2^{15}$ and switch to another values for resolution tests.

Time evolution of $\psi(x_j)$ is calculated by sequentially acting with operators in Eq. (S3). The action of the "kinetic" parts of the evolution operator $\mathrm{e}^{-ic_\alpha\Delta t\,\hat{p}^2/2}$ is performed as follows. First, we Fourier-transform the field, $\psi(x_j) = \sum_k \tilde{\psi}(p_k)\mathrm{e}^{-ip_kx_j}$, using Fast Fourier transform algorithm, where $\tilde{\psi}(p_k)$ is the image at discrete momenta $p_k = 2\pi k/L$, $-N/2 < k \leq N/2$. After that multiplication by "kinetic" part of the evolution operator corresponds to phase rotation $\tilde{\psi}(p_k) \to \tilde{\psi}(p_k)\,\mathrm{e}^{-ic_1\Delta t\, p_k^2/2}$. Then we return to the coordinate representation for the ψ with the inverse Fourier transform and multimply it by the "potential" part of the evolution operator. These actions are repeated the required number of times according to Eq. (S3).

Each step of the algorithm is reduced to multiplying the wave function by a phase, so it exectly (up to truncation errors) preserves the norm. The total energy is not conserved. However, we have chosen a time step such that the relative non-conservation of energy in our solutions has never exceeded 10^{-12}. This accuracy allows us to observe the appearance of wave packets with an amplitude up to 10^{-10}.

1. H. Yoshida, "Construction of higher order symplectic integrators," *Physics Letters A*, vol. 150, no. 5, pp. 262–268, 1990.
2. R. McLachlan, "Symplectic integration of Hamiltonian wave equations," *Numerische Mathematik*, vol. 66, no. 1, p. pages 465–492, 1993.