Supplementary Material to the article
"Comparison of two thulium optical clocks using synchronous interrogation"

I. MEASUREMENT CYCLE SEQUENCE

A single measurement cycle when operating thulium optical clock has a duration of $T_c = 1\,\mathrm{s}$ and consists of the following stages, schematically shown in Fig. SS1.

1. Loading of thulium atoms from a thermal atomic beam using Zeeman cooling is carried out within 500 ms.

2. Within 50 ms, the atoms in the first-stage magneto-optical trap (MOT) are cooled, and a cloud of atoms is formed in the center of the trap.

3. Within 50 ms, the atoms are cooled in the second-stage MOT, operating at a 530 nm transition with a natural width of 350 kHz, resulting in the formation of a cloud of atoms with a diameter of about 150 µm and atomic temperature of $10 - 20\,\mu\mathrm{K}$.

4. After the MOT is turned off, some of the atoms (about 30%) are trapped in the optical lattice, which is formed by radiation at a wavelength close to the magic wavelength of 1063.65 nm using an enhancement cavity. Together with turning off the MOT, the magnetic field is turned on to set the quantization axis. The magnetic field direction is perpendicular to the polarization vector of the optical lattice (see Fig. S1.c from the main text) because the magic wavelength near 1063.65 nm is realized for such configuration. Within 50 ms, the bias magnetic field is set to a stationary value, and atoms, which are not captured in the optical lattice, leave the detection region.

5. Optical pumping is performed to the $|g, F = 4, m_F = 0\rangle$ and $|F = 3, m_F = 0\rangle$ levels of the ground state using $\pi-$polarized radiation at wavelength 418 nm with duration 10 ms [21].

6. Then "sifting" of atoms is carried out by lowering the optical lattice depth [24] by $\tau \sim 20\,\mathrm{ms}$, as a result of which "hot" atoms fly out of the trapping potential of the optical lattice. The final depth of the optical lattice, at which spectroscopy of the clock transitions is subsequently carried out, is set in 20 ms.

7. Excitation of two clock transitions is carried out simultaneously by 80-ms pulses, each being generated by a separate acoustooptic modulator (AOM), shown schematically in Fig. S1.c from the main text. The frequencies of all AOMs are set by generators, referenced to the active hydrogen maser. The beams are combined on a polarization beam splitter and fed into a PM-single-mode optical fiber, which delivers the radiation to a vacuum chamber with atoms in the optical lattice.

8. 5 ms after the end of the clock pulses, the number of atoms at four levels is measured (at each of the two hyperfine states of the ground and clock levels), from which the excitation efficiency of each clock transition 4-3 and 3-2 is determined.

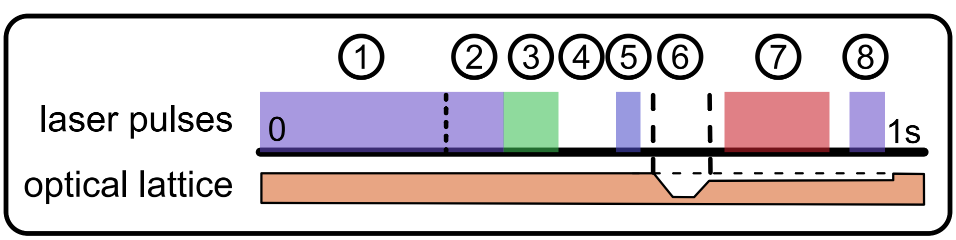

FIG. S1. Single measurement cycle scheme.