Supplementary Material to the article "Manifestation of layer-by-layer localization of van Hove singularities in tunneling between sheets of bilayer graphene"

SM 1. Device preparation

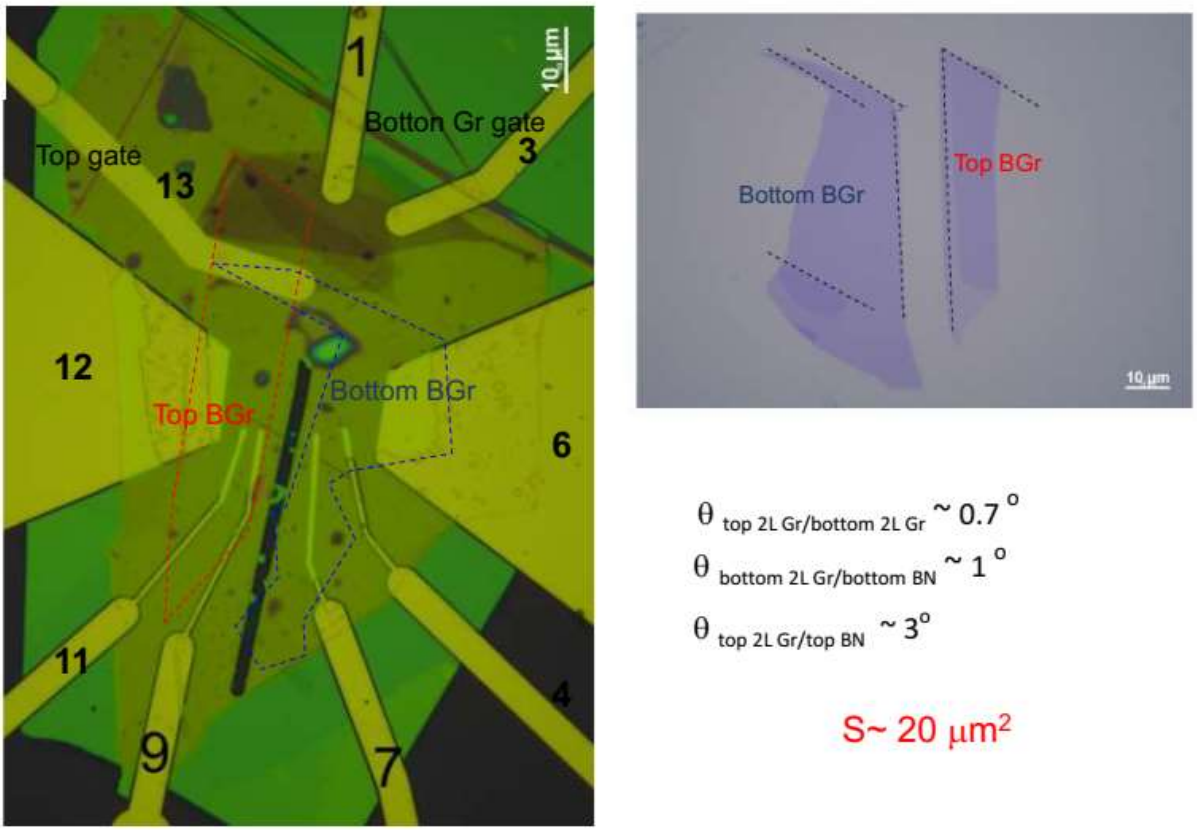

Fig. S1. Tunnelling devices were fabricated using the dry-transfer technique applied to the micro-mechanically cleaved layers of Bernal bilayer graphene, and *h*BN of various thicknesses (tunnelling layer, two encapsulating top and back graphene layers and back gate graphite electrode layers) from bulk graphite and *h*BN crystals, respectively. Those were assembled on top of each other into van der Waals heterostructures using PC (PolyBisphenol carbonate) stamps prepared on commercial PDMS (poly-dimethylsiloxane) films, and eventually, were deposited on top of 290 nm thick silicon dioxide/strongly p-doped silicon wafers. The twist angle of about 0.7° degrees was introduced unintentionally in-between the bottom and top bilayer graphene layers during the fabrication procedure. Next, Cr/Au edge contacts were made on the bottom and top Bernal bilayer graphene using electron-beam lithography followed by *h*BN etching, metal deposition and a lift-off process. *h*BN was etched in a reactive ion etching system using CHF_3 chemistry. Contacts were made in such a way to have a four-probe measurement geometry. The top encapsulating *h*BN layer was additionally covered by a Cr/Au pad at the cross-sectional area of top and bottom Bernal bilayer graphene of $S \approx 20\,\mu\mathrm{m}^2$, which served as a top gate electrode.

SM 2.

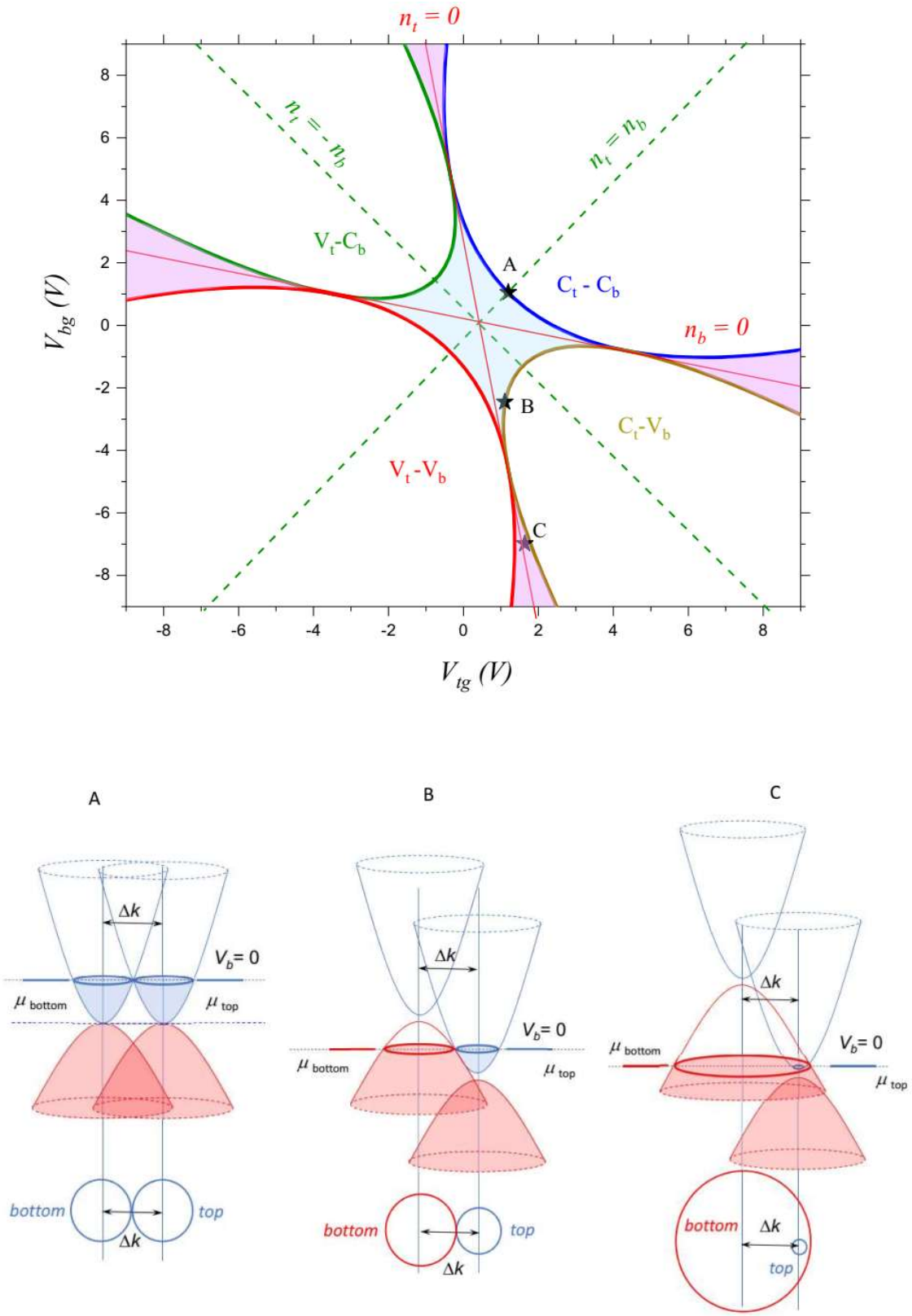

Fig. S2. Calculated map of tunneling boundaries in the experimental sample. The stars correspond to the schemes (A, B, C) of the mutual arrangement of zones in sheets of bilayer graphene

SM 3.

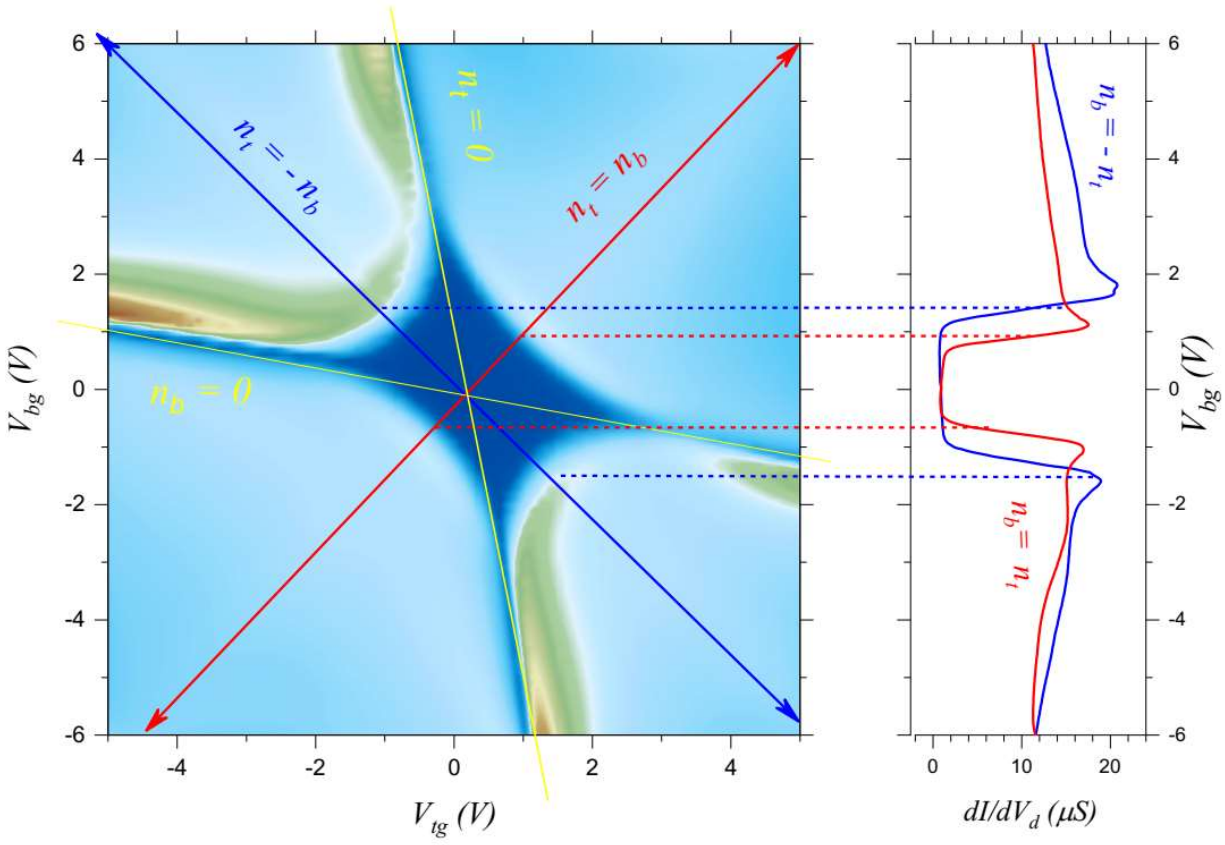

Fig. S3. Experimental conductivity map. The red and blue diagonals correspond to the conditions of equality of current carrier concentrations in the BLG flakes along which the map cuts are made, shown in the right panel.

SM 4.

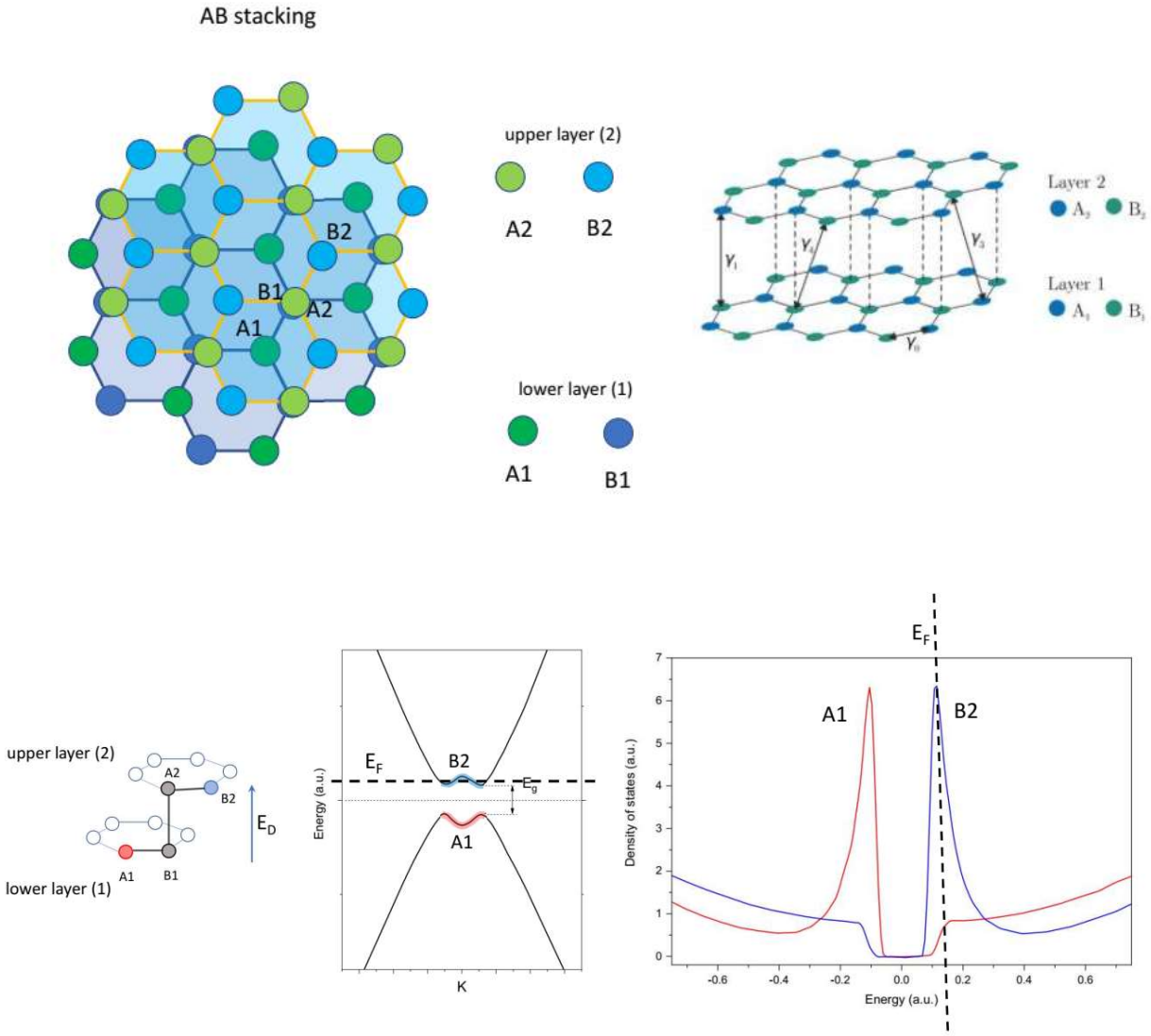

Fig. S4. *Top panel* - Lattice structure of Bernal bilayer graphene. Atoms A1 and B1 belong to different sublattices in one layer while atoms A2 and B2 belong to different sublattices in another layer.

Bottom panel - The electric field causes a local potential difference Δ between the layers, which breaks the inversion symmetry. The atomic orbitals of the A1 and B2 atoms (highlighted in red and blue) determine the low-energy spectra. The electron band structure of BLG near the K point and site-resolved density of states in biased bilayer graphene are shown [1-3].

SM 5.

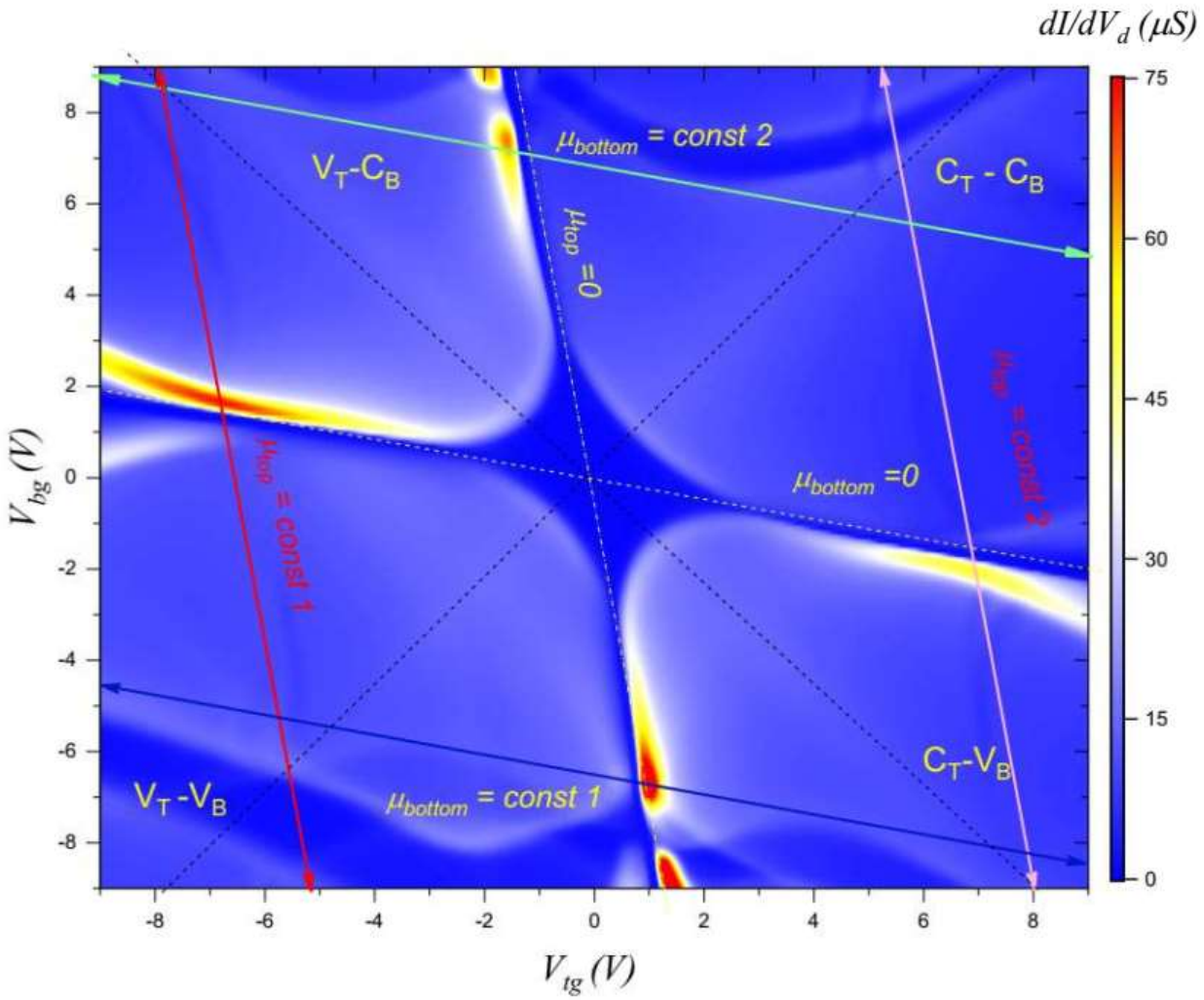

Fig. S5. Experimental conductivity map $dI/dV_d(V_{tg}; V_{bg})$, where the lobes C_t-C_b, V_t-V_b, etc. correspond to tunneling between states in the conducting and valence bands in the upper and lower BLG. Blue, green, red and pink lines on the map correspond to a constant carrier concentration in one of the layers, the cuts along which and the tunneling conditions are shown schematically below in Fig.S6-9.

SM 6.

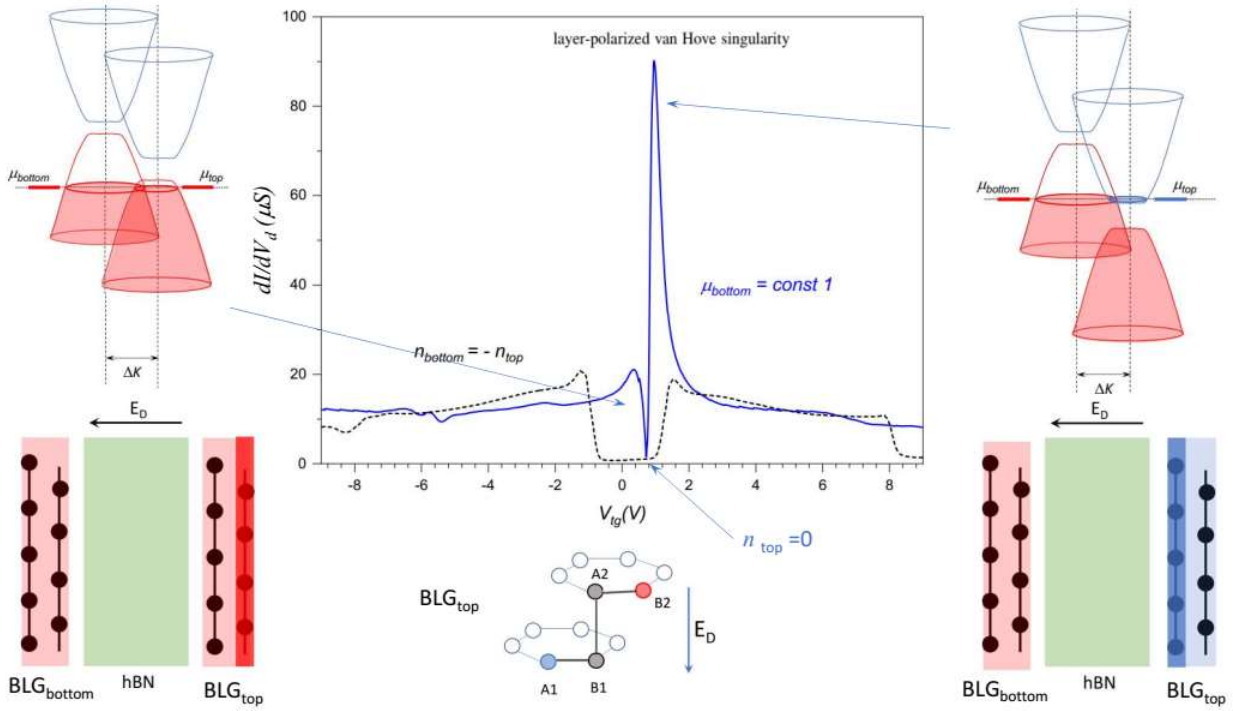

Fig. S6. Section of the SI6 map along the blue line, the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized far from the hBN barrier in the upper BLG (the localization is shown in intense red); the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized near the hBN barrier in the upper BLG (shown in intense blue).

SM 7.

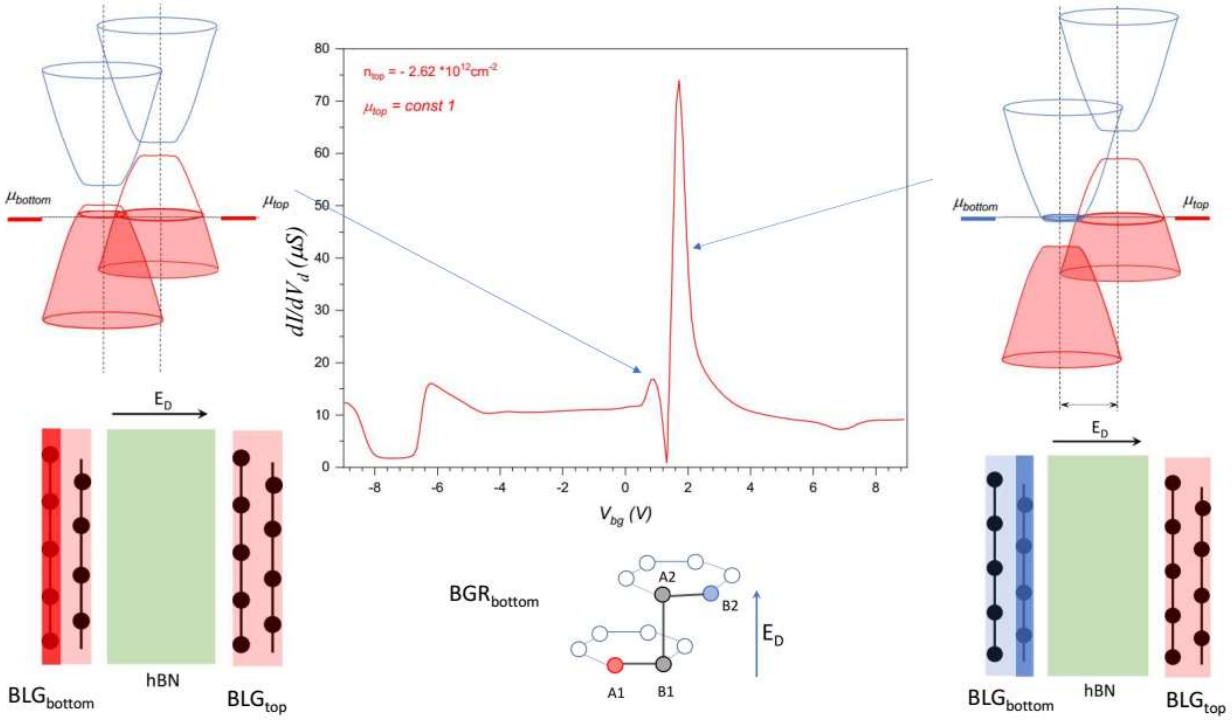

Fig. S7. Section of the SI6 map along the red line, the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized far from the hBN barrier in the upper BLG (the localization is shown in intense red); the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized near the hBN barrier in the upper BLG (shown in intense blue).

SM 8.

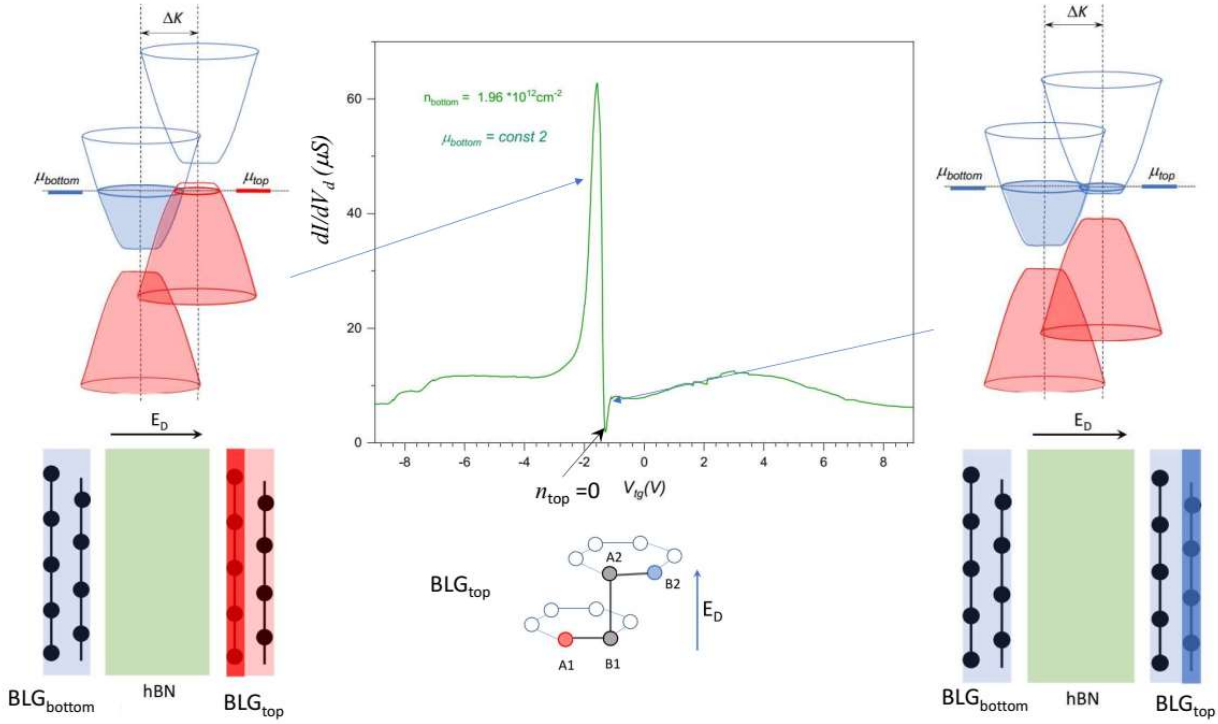

Fig. S8. Section of the SI6 map along the green line, the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized far from the hBN barrier in the upper BLG (the localization is shown in intense red); the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized near the hBN barrier in the upper BLG (shown in intense blue).

SM 9.

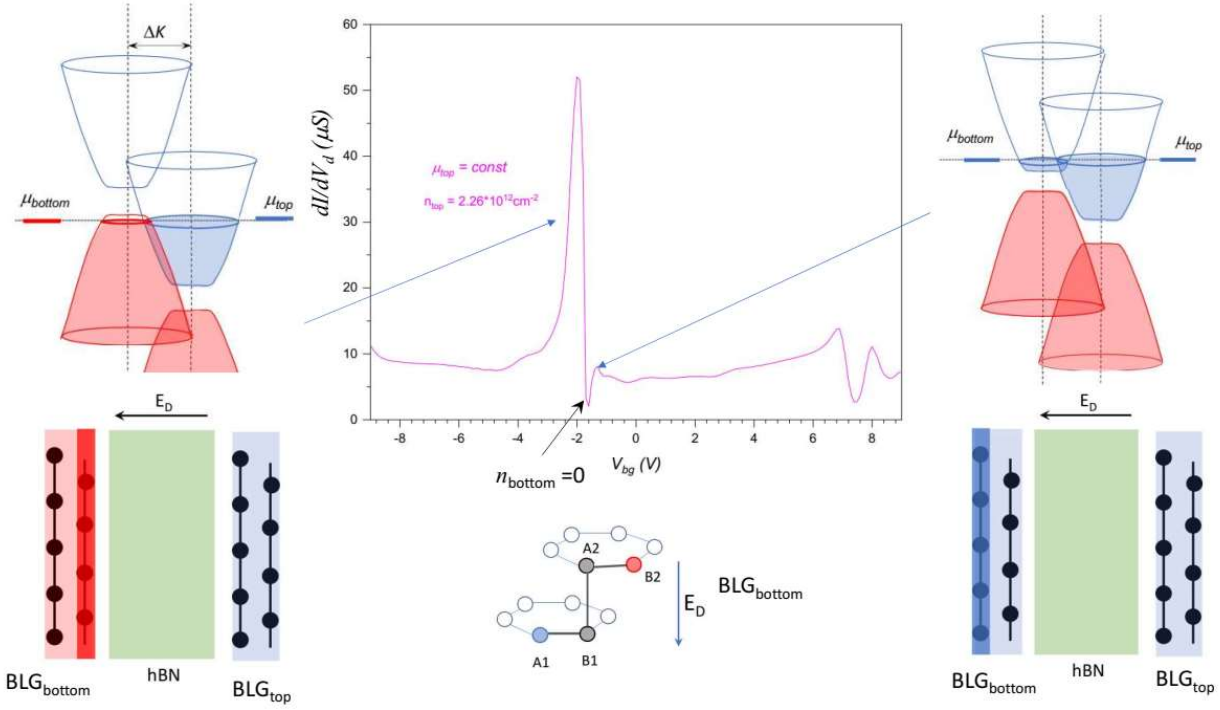

Fig. S9. Section of the SI6 map along the pink line, the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized far from the hBN barrier in the upper BLG (the localization is shown in intense red); the relative position of the zones and the scheme of the layers of the structure during tunneling with the participation of the van Hove singularity localized near the hBN barrier in the upper BLG (shown in intense blue).

[1] Ashwin Ramasubramaniam, Tunable Band Gaps in Bilayer Graphene−BN Heterostructures, Nano Letters, Vol 11, Issue 3, 1070, (2011).

[2] Keun Su Kim, Tae-Hwan Kim, Andrew L. Walter, Thomas Seyller, Han Woong Yeom, Eli Rotenberg, and Aaron Bostwick, Visualizing Atomic-Scale Negative Differential Resistance in Bilayer Graphene, Phys. Rev. Lett. 110, 036804, (2013).

[3] Eike Icking, Luca Banszerus, Frederike Wörtche, Frank Volmer, Philipp Schmidt, Corinne Steiner, Stephan Engels, Jonas Hesselmann, Matthias Goldsche, Kenji Watanabe, Takashi Taniguchi, Christian Volk, Bernd Beschoten, Christoph Stampfer, Transport Spectroscopy of Ultraclean Tunable Band Gaps in Bilayer Graphene, Adv. Electron. Mater., 2200510, (2022).