Supplementary Material to the article “Photon drag at the junction between metal and 2d semiconductor”

I. FACTORIZED DIELECTRIC FUNCTIONS

Factorization of the dielectric function $\varepsilon(q) = \varepsilon_{-}(q)\varepsilon_{+}(q)$ is achieved by applying the Cauchy theorem of complex analysis to a narrow strip of width 2γ enclosing the real q-axis. This result is

$$\varepsilon_{\pm}(q) = \exp\left\{ \pm\frac{1}{2\pi i}\int\limits_{-\infty}^{+\infty} \frac{\ln\varepsilon(u)\,du}{u-(q\pm i\gamma)} \right\}. \tag{S1}$$

It applies to both dielectric functions in the s- and p-polarizations. It is possible to relate the factorized dielectric functions in the two polarizations using an algebraic identity

$$1+\eta\frac{\sqrt{k_0^2-q^2}}{k_0} = \frac{\eta k_0}{\sqrt{k_0^2-q^2}}\left(1+\frac{1}{\eta}\frac{\sqrt{k_0^2-q^2}}{k_0}\right). \tag{S2}$$

Applying the factorization procedure to both sides, we get

$$\varepsilon^{\mathrm{s}}_{\pm}(q,\eta) = \frac{\sqrt{\eta k_0}}{\sqrt{k_0 \pm q}}\varepsilon^{\mathrm{p}}_{\pm}(q,\eta^{-1}). \tag{S3}$$

It implies that only one non-trivial factorization is required, say, for $\varepsilon^{\mathrm{p}}_{\pm}(q)$. Analytical evaluation of $\varepsilon^{\mathrm{p}}_{\pm}(q)$ via dilogarithm functions is possible and is presented in [1]. Alternatively, a direct numerical evaluation using Eq. (S1) is possible. When dealing with q-values on the real axis (which is sufficient for all calculations), we rewrite Eq. S1 as:

$$\varepsilon_{\pm}(q) = \sqrt{\varepsilon(q)} \times \exp\left\{ \pm\int\limits_{0}^{+\infty} \frac{\ln\varepsilon(q+v)-\ln\varepsilon(q-v)}{2\pi i v}dv \right\}. \tag{S4}$$

The convenience of representation (S4) stems from removal of integration singularities at zero and infinity. The integral converges in an ordinary sense (not only as principal value), and simple grid integration methods can be applied.

II. MOMENTUM TRANSFER FROM FERMI GOLDEN RULE

From quantum perspective, momentum transfer form field to electrons and holes is realized either upon intraband or upon interband absorption of light quanta with momentum $\mathbf{q}$ and energy $\hbar\omega$. The rate of momentum transfer $\partial\mathbf{P}/\partial t$ can be obtained by multiplying the momentum transfer upon individual absorption event $\Delta\mathbf{p}$ by the Fermi golden rule transition probability W and integrating over the phase space. For definiteness, we perform the calculation for monolayer graphene, yet it can be easily generalized to other two-band semiconductors. The momentum transfer to the electronic subsystem is given by

$$\frac{\partial\mathbf{P}_e}{\partial t} = \left(\frac{\partial\mathbf{P}_e}{\partial t}\right)_{\mathrm{intra}} + \left(\frac{\partial\mathbf{P}_e}{\partial t}\right)_{\mathrm{inter}}, \tag{S5}$$

where the first and the second terms correspond to the intraband and interband transitions respectively. The band-resolved momentum transfer rates are given by

$$\left(\frac{\partial\mathbf{P}_e}{\partial t}\right)_{\mathrm{intra}} = \frac{2\pi}{\hbar}N\sum_{p}\mathbf{q}\left[f_c(\mathbf{p}_-)-f_c(\mathbf{p}_+)\right]\left|\langle c\mathbf{p}_-\left|\hat{V}_{em}\right|c\mathbf{p}_+\rangle\right|^2\delta\left(\hbar\omega+\varepsilon_{\mathbf{p}-}-\varepsilon_{\mathbf{p}+}\right), \tag{S6}$$

$$\left(\frac{\partial\mathbf{P}_e}{\partial t}\right)_{\mathrm{inter}} = \frac{2\pi}{\hbar}N\sum_{p}\left(\mathbf{p}+\frac{\mathbf{q}}{2}\right)\left[f_v(\mathbf{p}_-)-f_c(\mathbf{p}_+)\right]\left|\langle v\mathbf{p}_-\left|\hat{V}_{em}\right|c\mathbf{p}_+\rangle\right|^2\delta\left(\hbar\omega-\varepsilon_{\mathbf{p}+}-\varepsilon_{\mathbf{p}-}\right). \tag{S7}$$

In the above equations, transitions occur due to the electromagnetic interaction of electrons with field governed by the Hamiltonian

$$\hat{V}_{em} = \frac{e}{c}\hat{\mathbf{v}}_{\mathbf{p}}\mathbf{A}, \tag{S8}$$

where $\hat{\mathbf{v}}_{\mathbf{p}}$ is the electron velocity operator, $\mathbf{A}$ is the field vector potential. In Eqs. (S6) and (S7) we have introduced initial and final momenta $\mathbf{p}_{\pm} = \mathbf{p} \pm \mathbf{q}/2$, the conduction and valence band distribution functions f_c and f_v, and the degeneracy factor N.

There key difference between intra- and interband momentum transfer lies in the following. In the intraband process, the momentum is changed by $\Delta\mathbf{p} = \mathbf{q}$ which can be factored out of the integration sign. As a result, the momentum transfer becomes proportional to the ordinary electronic intraband conductivity $\mathrm{Re}\sigma_{\mathrm{e}}$:

$$\left(\frac{\partial \mathbf{P}_e}{\partial t}\right)_{\mathrm{intra}} = \frac{\mathbf{q}}{\hbar\omega}\left(\frac{\partial \mathcal{E}_e}{\partial t}\right)_{\mathrm{intra}} = 2\frac{\mathbf{q}}{\hbar\omega}\mathrm{Re}\sigma_{\mathrm{e}}|\mathbf{E}_{\omega\mathbf{q}}|^2. \tag{S9}$$

The first equality represents the relation between momentum and energy transfer, while the second one represents the Joule's law for electromagnetic absorption. The microscopic expression for $\mathrm{Re}\sigma_{\mathrm{e}}$ obtained by applying the line of transformations (S9) to Eq. (S6) agrees with Kubo formula [2] and reads as:

$$\mathrm{Re}\sigma_{\mathrm{e}} = \pi N\frac{e^2}{\omega}\int\frac{d\mathbf{p}}{(2\pi\hbar)^2}\left[f_c\left(\mathbf{p}_-\right) - f_c\left(\mathbf{p}_+\right)\right]\left|\langle c\mathbf{p}_-\left|\hat{\mathbf{v}}\right| c\mathbf{p}_+\rangle\right|^2\delta\left(\hbar\omega + \varepsilon_{\mathbf{p}-} - \varepsilon_{\mathbf{p}+}\right). \tag{S10}$$

In the interband process, an electron in conduction band is generated with initial momentum $\Delta\mathbf{p} = \mathbf{q}/2 + \mathbf{p}$. The term with $\mathbf{q}/2$ yields half of an ordinary interband conductivity. The term with $\mathbf{p}$ is anomalous and is responsible for the 'resonant photon drag' considered in [3]:

$$\left(\frac{\partial \mathbf{P}_e}{\partial t}\right)_{\mathrm{inter}} = \frac{\mathbf{q}}{\hbar\omega}\mathrm{Re}\sigma_{\mathrm{inter}}|\mathbf{E}_{\omega\mathbf{q}}|^2 + \left(\frac{\partial \mathbf{P}_e}{\partial t}\right)_{\mathrm{res}}, \tag{S11}$$

$$\left(\frac{\partial \mathbf{P}_e}{\partial t}\right)_{\mathrm{res}} = \frac{2\pi}{\hbar}N\sum_{p}\mathbf{p}\left[f_v\left(\mathbf{p}_-\right) - f_c\left(\mathbf{p}_+\right)\right]\left|\langle v\mathbf{p}_-\left|\hat{V}_{em}\right| c\mathbf{p}_+\rangle\right|^2\delta\left(\hbar\omega - \varepsilon_{\mathbf{p}+} - \varepsilon_{\mathbf{p}-}\right) \tag{S12}$$

The ordinary interband conductivity is given by

$$\mathrm{Re}\sigma_{\mathrm{inter}} = \pi N\frac{e^2}{\omega}\int\frac{d\mathbf{p}}{(2\pi\hbar)^2}\left[f_v\left(\mathbf{p}_-\right) - f_c\left(\mathbf{p}_+\right)\right]\left|\langle v\mathbf{p}_-\left|\hat{\mathbf{v}}\right| c\mathbf{p}_+\rangle\right|^2\delta\left(\hbar\omega - \varepsilon_{\mathbf{p}-} - \varepsilon_{\mathbf{p}+}\right). \tag{S13}$$

The resonant interband photon drag is proportional neither to $\mathrm{Re}\sigma_{\mathrm{intra}}$ nor to $\mathrm{Re}\sigma_{\mathrm{inter}}$. Still, it is possible to see that expansion of $(\partial\mathbf{P}_e/\partial t)_{\mathrm{res}}$ in powers of $\mathbf{q}$ starts from the linear term. This facts suggests to introduce the 'resonant interband' conductivity $\mathrm{Re}\sigma_{\mathrm{res}}$ such that

$$\left(\frac{\partial \mathbf{P}_e}{\partial t}\right)_{\mathrm{res}} = 2\frac{\mathbf{q}}{\hbar\omega}\mathrm{Re}\sigma_{\mathrm{res}}|\mathbf{E}_{\omega\mathbf{q}}|^2 \tag{S14}$$

The resonant conductivity would be fiven by

$$\mathrm{Re}\sigma_{\mathrm{res}} = -\pi\hbar\omega N\frac{e^2}{\omega}\int\frac{d\mathbf{p}}{(2\pi\hbar)^2}\mathbf{p}\frac{\partial\varepsilon_{\mathbf{p}}}{\partial\mathbf{p}}\left[\frac{\partial f_v}{\partial\varepsilon} + \frac{\partial f_c}{\partial\varepsilon}\right]\left|\langle v\mathbf{p}_-\left|\hat{\mathbf{v}}\right| c\mathbf{p}_+\rangle\right|^2\delta\left(\hbar\omega - \varepsilon_{\mathbf{p}-} - \varepsilon_{\mathbf{p}+}\right). \tag{S15}$$

Collecting the terms responsible for the change in electron momentum, we get

$$\left(\frac{\partial \mathbf{P}_e}{\partial t}\right) = 2\frac{\mathbf{q}}{\hbar\omega}\mathrm{Re}\left[\sigma_{\mathrm{e}} + \sigma_{\mathrm{inter}}/2 + \sigma_{\mathrm{res}}\right]\left|\mathbf{E}_{\omega\mathbf{q}}\right|^2. \tag{S16}$$

A similar line of transforms for the valence band results in

$$\left(\frac{\partial \mathbf{P}_h}{\partial t}\right) = 2\frac{\mathbf{q}}{\hbar\omega}\mathrm{Re}\left[\sigma_{\mathrm{h}} + \sigma_{\mathrm{inter}}/2 - \sigma_{\mathrm{res}}\right]\left|\mathbf{E}_{\omega\mathbf{q}}\right|^2. \tag{S17}$$

We further proceed to the case of slowly varying fields. In this case, all expressions for conductivity can be evaluated at $\mathbf{q} = 0$. In the particular case of graphene, we find

$$\mathrm{Re}\sigma_{\mathrm{inter}} = \frac{e^2}{4\hbar}\left[f_v(-\hbar\omega/2) - f_c(-\hbar\omega/2)\right], \tag{S18}$$

$$\mathrm{Re}\sigma_{\mathrm{res}} = \frac{e^2}{4\hbar}\frac{\hbar\omega}{16kT}\frac{\sinh\frac{\hbar\omega}{2kT} + \sinh\frac{\varepsilon_F}{kT}}{\left(\cosh\frac{\hbar\omega}{2kT} + \cosh\frac{\varepsilon_F}{kT}\right)^2}, \tag{S19}$$

$$\mathrm{Re}\sigma_{\mathrm{e}} = i\frac{e^2}{\pi\hbar}\frac{kT\ln\left(1+e^{\varepsilon_F/kT}\right)}{\hbar(\omega + i/\tau_{p,e})}, \tag{S20}$$

$$\mathrm{Re}\sigma_{\mathrm{h}} = i\frac{e^2}{\pi\hbar}\frac{kT\ln\left(1+e^{-\varepsilon_F/kT}\right)}{\hbar(\omega + i/\tau_{p,h})}. \tag{S21}$$

In the same limit of slowly varying fields, we can pass to the local representation for momentum transfer by writing $\mathbf{q}/\hbar = -i\partial/\partial\mathbf{r}$. This leads us to the final expressions for the forces:

$$\mathbf{f}_e \equiv \left(\frac{\partial \mathbf{P}_e}{\partial t}\right) = \frac{2}{\omega}\mathrm{Im}\left\{(\sigma_{\mathrm{e}} + \sigma_{\mathrm{inter}}/2 + \sigma_{\mathrm{res}})\left[\mathbf{e}_x\left(E_x^*\partial_x E_x + E_y^*\partial_x E_y\right) + \mathbf{e}_y\left(E_x^*\partial_y E_x + E_y^*\partial_y E_y\right)\right]\right\}, \tag{S22}$$

$$\mathbf{f}_h \equiv \left(\frac{\partial \mathbf{P}_h}{\partial t}\right) = \frac{2}{\omega}\mathrm{Im}\left\{(\sigma_{\mathrm{h}} + \sigma_{\mathrm{inter}}/2 - \sigma_{\mathrm{res}})\left[\mathbf{e}_x\left(E_x^*\partial_x E_x + E_y^*\partial_x E_y\right) + \mathbf{e}_y\left(E_x^*\partial_y E_x + E_y^*\partial_y E_y\right)\right]\right\} \tag{S23}$$

[1] E. Nikulin, D. Mylnikov, D. Bandurin, and D. Svintsov, Edge diffraction, plasmon launching, and universal absorption enhancement in two-dimensional junctions, Physical Review B **103**, 085306 (2021).

[2] L. A. Falkovsky and A. A. Varlamov, Space-time dispersion of graphene conductivity, European Physical Journal B **56**, 281 (2007).

[3] M. V. Entin, L. I. Magarill, and D. L. Shepelyansky, Theory of resonant photon drag in monolayer graphene, Phys. Rev. B **81**, 165441 (2010).