Supplementary Material to the article "Magnetoelectric non-reciprocity effect in submillimetre EPR spectroscopy of $LiYF_4$ doped with $^{166}Er^{3+}$ ions"

As the laboratory coordinate system (see Fig. 2 of the main text) we take the triple vectors of the electromagnetic wave $\boldsymbol{E}_1$, $\boldsymbol{H}_1$, $\boldsymbol{k}$. The rotation axis of the sample is parallel to the vector $\boldsymbol{k}$. The external magnetic field $\boldsymbol{H}$ is perpendicular to $\boldsymbol{k}$.

In the laboratory system we have the following set of vector components: (0, E_1,0), (H_1, 0,0) , (0, H,0) and (0,0,-k). The crystal field operator and the energy operators of interaction with the electromagnetic wave are written in the system of crystallographic axes a,b,c. To calculate the wave functions and transition probabilities, it is necessary to have the values of the components of the vectors $\boldsymbol{H}_1$, $\boldsymbol{E}_1$, $\boldsymbol{H}$ in the system of axes a,b,c .

Successive trasformation of coordinate systems using Euler angles yields the following relations

$H_c= H \sin\varphi \sin\vartheta$,

$H_a = H \sin\varphi \cos\vartheta$,

$H_b = H \cos \varphi$,

$H_{1c}=H_1 \cos\varphi\sin\vartheta$,

$H_{1a} =H_1 \cos\varphi\cos\vartheta$,

$H_{1b} =-H_1 \sin\varphi$,

$E_{1c} =E_1 \sin\varphi \sin\vartheta$,

$E_{1a} =E_1 \sin\varphi \cos\vartheta$,

$E_{1b} =E_1 \cos\varphi$.

Here ϑ is the angle between the wave vector $\boldsymbol{k}$ and the c-axis of the crystal. The angle φ describes the rotation of the sample around the direction of the wave vector.