Supplementary Material to the article “Lateral plasmonic superlattice in strongly dissipative regime”

Here we present analytical formulas describing the various regimes for the case of intermediate coupling $|s_1-s_2|/s_1 \sim 1$, which was discussed in the main body of the paper in the Section 5. Here, we neglect contribution of small h_n putting $h_n = 0$.

Different regimes of dissipation of the lateral plasmonic crystal (LPC) can be illustrated in the diagram shown in Fig. S1 (see also Fig. 3b in the main body of the paper). Below, we present analytical equations for dissipation in these regimes. All these equations can be derived from the general equation (19). It is worth noting that diagram Fig. S1 is more complicated than the corresponding diagram of a single field effect transitor (FET). Indeed, single FET has one resonant frequency ω_0 and for $\omega_0 \ll \gamma$ possible excitation regimes are the short cell regime and the long cell regime (including high frequency limit). Plasmonic crystal in strong coupling case ($L_1^* \gg L_2^*$) supports additional intermediate regime, when $L_1^* \gg L_1$ and at the same time $L_2^* \ll L_2$ (compare Fig. S1 with different regimes of single FET excitation, see Fig. 1 of [45]).

“Short cell” regime

First, we discuss *“short cell” regime* (see region I at Fig. S1 and Fig. 3b). In the limit of very small frequencies we have

$$\Omega \approx \Gamma \approx \sqrt{\omega\gamma/2}, \;\; \Sigma \approx \frac{\sqrt{2}(1-i)}{\sqrt{\omega\gamma}}\left(\frac{s_1^2}{L_1}+\frac{s_2^2}{L_2}\right), \quad \text{(S1)}$$

and $L_1^* \gg L_1$, $L_2^* \gg L_2$. Using these formulas, one can expand Eq. (19) over ω up to the second order. The results reads (for simplicity we take $L_1 = L_2 = L/2$):

$$P = P_0\left[\frac{2s_1s_2}{s_1^2+s_2^2}\right]^2\left[1+\omega^2\xi\frac{\gamma^2(1-k^2)^2}{\omega_2^4}-\frac{\omega^2}{\gamma^2}\right]. \quad \text{(S2)}$$

Here $k = s_2/s_1$ ($0 \le k \le 1$) and $\xi = \pi^4(k^4 + 22k^2+1)/2880(1+k^2)^2$ ($0.03 < \xi < 0.21$). The first term in the square brackets corresponds to the static Ohm's law, while term $\propto \gamma^{-2}$ comes from the Drude factor: $\gamma^2/(\gamma^2+\omega^2) \approx 1-\omega^2/\gamma^2$. It worth noting that dissipation at small frequency strongly depends on type of LPC coupling: $P(0) \approx P_0$ for weak coupling ($s_1 \approx s_2$), while for strong coupling ($s_1 \gg s_2$), dissipation is much lower due to blocking of dc current in the zero-frequency limit by low-conducting passive regions: $P(0) \approx P_0 s_2^2/s_1^2$.

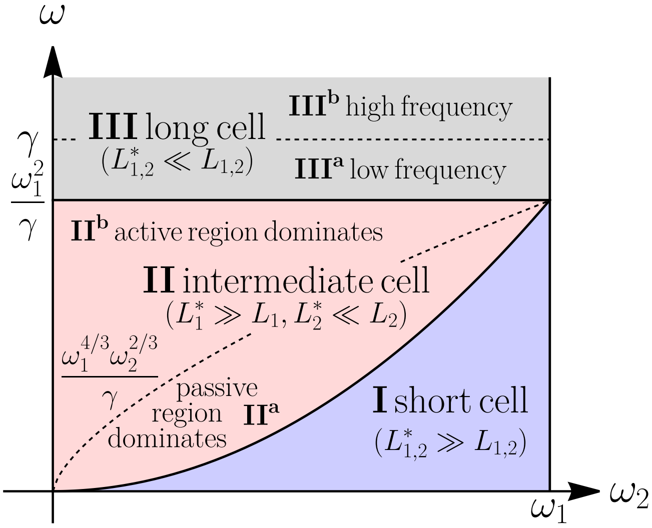

FIG. S1. Different regimes of dissipation in non-resonant LPC ($\gamma \gg \omega_{1,2}$) schematically presented on a (ω, ω_2)-plane while γ and ω_1 are fixed and $\gamma \gg \omega_1 \ge \omega_2$. Here we assume $L_1 \sim L_2$ and omit numerical coefficients. Thick lines divide diagram into three regimes: (I) short cell, (II) intermediate cell and (III) long cell. Condition $L_2^* = L_2$ defines boundary between regimes (I) and (II) ($\omega = \omega_2^2/\gamma$). Analogically, line between (II) and (III) is found from $L_1^* = L_1$ ($\omega = \omega_1^2/\gamma$). Dashed line $\omega = \gamma$ splits regime (III) into high and low frequency regimes. Dashed line $\omega^3 = \omega_1^4\omega_2^3/\gamma^3$ separates region II into sub-regions (II^b) and (II^a), where dominant contribution to the dissipation is given by plasmonic oscillations in the active region or exponentially decaying tails in the passive regions, respectively.

The term $\propto \xi$ appears due to finite time of the Maxwell relaxation in the passive stripe. We see that there are two corrections, which are proportional to ω^2 and have different signs. For $s_1 \sim s_2$ ($k \sim 1$) the term $\propto \xi$ is larger and the ω^2-correction is positive. However, in the weak coupling regime, when k turns to unity, the ω^2-correction can change sign. Assuming now arbitrary

L_1 and L_2 and denoting $\delta s = s_1 - s_2$, for weak coupling regime $\delta s \ll s_1$, the relative correction is

$$\frac{P(\omega) - P(0)}{P(0)} = -\frac{\omega^2}{\gamma^2} + \frac{\delta s^2 \gamma^2 L_1^2 L_2^2 (L_1^2 + 4L_1 L_2 + L_2^2)}{180(L_1 + L_2)^2 s_1^6}\omega^2. \tag{S3}$$

For $L_1 = L_2 = L/2$ change of sign happens for the following value of the plasma wave velocity modulation: $\delta s / s_1 = 2\sqrt{30}\omega_1^2/\gamma^2\pi^3 \approx 1.1\ \omega_1^2/\gamma^2$.

"Intermediate cell" regime

Let us now discuss *"intermediate cell" regime* (see region II at Fig. S1 and Fig. 3b). In this regime,

$$\Omega \approx \Gamma \approx \sqrt{\omega\gamma/2},\ \ \Sigma \approx -is_2 + \frac{(1-i)s_1^2}{\sqrt{\omega\gamma}L_1}, \tag{S4}$$

$L_2^* \ll L_2$, $L_1^* \gg L_1$, and plasma waves existed in the active regions exponentially decay into region "2" from the inter-region boundaries. Frequency conditions for the intermediate cell regime read $\omega_2 \ll \sqrt{\omega\gamma} \ll \omega_1$. Keeping in mind that $\omega \ll \gamma$, we find from Eq. (19)

$$\begin{aligned} P = {} & P_0 \frac{\pi^4}{120} \frac{\omega^2\gamma^2 L_1}{\omega_1^4 (L_1 + L_2)} \\ & + P_0 \left[\frac{(2L_1 + L_2)s_2^2}{L_1 s_1^2 + L_2 s_2^2} + \frac{\sqrt{\omega\gamma} L_1^2 s_2}{2\sqrt{2}(L_1 s_1^2 + L_2 s_2^2)} \right] \end{aligned} \tag{S5}$$

First term, proportional to ω^2, represents contribution to the dissipation from active region, where $L_1^* \gg L_1$. Physically, this term appears due to finite time of the Maxwell relaxation in the active region. The frequency-dependent term, $\propto \sqrt{\omega}$, in the square brackets describes dissipation within boundary regions on the order L_2^* in the passive regions. These two terms compete with each other, so that either passive or active region can dominate in the dissipation. Equalizing the terms proportional to ω^2 and $\sqrt{\omega}$, and for $L_1 = L_2 = L/2$ we get

$$\omega = \omega^* = \left(\frac{1800\, \omega_1^4 \omega_2^2}{\pi^6 \gamma^3} \right)^{1/3} \approx 1.24\, \frac{\omega_1^{4/3} \omega_2^{2/3}}{\gamma}.$$

For $\omega \gg \omega^*$ the term, proportional to ω^2, is larger, so that dissipation in active region gives the leading contribution (see region II$^{\mathbf{b}}$ at Fig. S1 and Fig. 3b). The contribution of passive regions can be neglected and the system without loss of generality can be considered as an array of active strips separated by insulating regions. For smaller frequency ($\omega \ll \omega^*$) the $\sqrt{\omega}$-term dominates. This contribution comes from exponentially decaying plasma waves in region "2" (see region II$^{\mathbf{a}}$ at Fig. S1 and Fig. 3b).

"Long cell" regime

Next, we analyze *"long cell" regime*, (see region III at Fig.S1 and Fig. 3b). In this case, plasma waves exponentially decay in both active and passive regions: $L_{1,2}^* \ll L_{1,2}$, i.e. $\omega \gg \omega_{1,2}^2/\gamma$. Physically this means, that in both regions time of the Maxwell relaxation becomes larger than the period of the external field oscillation. The dissipation in this case is given by the following equation

$$P = \frac{P_0 \gamma^2}{\omega^2 + \gamma^2} \left[1 - \frac{(s_1 - s_2)^2 (s_1 + s_2)(3\Gamma^2 - \Omega^2)}{\Gamma(\Gamma^2 + \Omega^2)(L_1 s_1^2 + L_2 s_2^2)} \right], \tag{S6}$$

which is the product of the Drude dissipation in the homogeneous external field and a factor in square brackets, which slowly increases with ω due to correction caused by finite length of the boundary layer. The latter is negative at relatively low frequency and saturates at positive value for very high frequencies, where ω becomes larger than γ, so that according to Eq. (15) plasmon damping length saturates on the value s_1/γ for $\omega \to \infty$. Hence, there are two regions III$^{\mathbf{a}}$ and III$^{\mathbf{b}}$ at Figs. S1 and 3b, which we will discuss in detail below.

RELATIVELY LOW FREQUENCY

We focus now at *relatively low frequency*, region III$^{\mathbf{a}}$ at Fig. S1 and Fig. 3b. This regime was briefly discussed in [34] (see Eq.(55) there). For $\omega_1^2/\gamma \ll \omega \ll \gamma$, we get

$$\Omega \approx \Gamma \approx \sqrt{\omega\gamma/2},\ \ \Sigma \approx -i(s_2 + s_2) \tag{S7}$$

and Eq. (S6) simplifies:

$$P \approx P_0 \left[1 - \frac{C}{\sqrt{\omega}} - \frac{\omega^2}{\gamma^2} \right] \tag{S8}$$

with

$$C = \frac{\sqrt{2}(s_1 - s_2)^2 (s_1 + s_2)}{\sqrt{\gamma}(L_1 s_1^2 + L_2 s_2^2)}.$$

Two frequency-dependent terms in the square brackets represent corrections, competition between which yields the maximum of P (see Fig. 3). For $s_2 \ll s_1$, dissipation is maximal at $\omega \sim \omega_1^{2/5}\gamma^{3/5}$.

Introducing $L_{\text{eff}} = (L_1 s_1^2 + L_2 s_2^2)/s_1^2$, one can rewrite Eq. (S8) as follows

$$P = P_0 \left[1 - \left(1 - \frac{s_2}{s_1} \right)^2 \frac{L_1^* + L_2^*}{L_{\text{eff}}} - \frac{\omega^2}{\gamma^2} \right]. \tag{S9}$$

We note that in the isolated strip ($s_2 = 0$) dissipation is $P = P_0(1 - L_1^*/L_1 - \omega^2/\gamma^2)$, and in the homogeneous system ($s_2 = s_1$) dissipation is given by the Drude formula.

HIGH FREQUENCY REGIME

Next, we focus on *high frequency*, region III$^{\mathbf{b}}$ at Fig. S1 and Fig. 3b. When external frequency exceeds γ, damping of plasma waves Γ saturates:

$$\Omega \approx \omega, \;\; \Gamma \approx \gamma/2, \;\; \Sigma \approx -i(s_2 + s_2), \tag{S10}$$

and Eq. (S6) becomes

$$P = \frac{\gamma^2}{\omega^2} P_0 \left[1 + \frac{2(s_1 - s_2)^2(s_1 + s_2)}{\gamma(L_1 s_1^2 + L_2 s_2^2)} \right]. \tag{S11}$$

The second term in the square brackets is small frequency-independent correction which comes from the narrow regions of width $s_{1,2}/\gamma$ near the boundaries. It is worth noting that this correction has opposite sign to $1/\sqrt{\omega}$ – correction in Eq. (S8).