

Supplementary Material to the article "New Method for Investigating the Presence of Extragalactic Magnetic Fields"

1. BACKGROUND INHOMOGENEITIES: INVESTIGATION USING MOCK BLAZARS

At the low brightness end of the distribution Fig. 5 of the main paper, measured offsets exceed the expected Extragalactic Magnetic Field (EGMF) effect. In this section, using mock blazars (implants), we show that this is due to the influence of background inhomogeneities.

At the lowest brightness level, even the limited statistics dominate, therefore, we cut out the dimmest part of the distribution at $\text{Log}(F) = -9.5$, the remaining sample includes 149 objects. The resulting χ^2 for this sample against null hypothesis (no EGMF effect and Gaussian homogeneous background) is 499 per 298 degrees of freedom (two degrees per each object for two coordinates). This means that the null hypothesis can be rejected with high significance ($p < 10^{-6}$) and the question arises, what is the reason for the high χ^2 : the echo effect or the background inhomogeneities?

There is a number of significant ($p < 10^{-5}$) offsets for individual objects if we use statistical RMS deviation estimated with randomization of azimuthal angle of photons around the source location. However, visual inspection shows that some of the outliers can be explained by interfering sources that passed through our filter described in the main paper. Can the "bad" background alone explain the high χ^2 value, or should we assume that this result is partly due to the cascade echo effect?

Although we have cleared the sample of objects with interfering sources from the 4FGL catalog and avoided areas with high background, there may still be unidentified faint sources and a small diffuse background gradient remaining. Probably, the only way to estimate the effect of background is to plant randomly to the data simulated photons from mock blazars. As a template, we choose 3C 454.3 for its brightness. Then we randomly selected photons from the template with their deviation from the centre and random azimuthal angle. The brightness distribution of implants was the same as of our sample of 149 real objects. We set 2000 implants isotropically, 730 of them passed the same low background cut that we used for real BL Lacs. After applying the same nearby source filter as for real BL Lacs, the implant sample was reduced to 350.

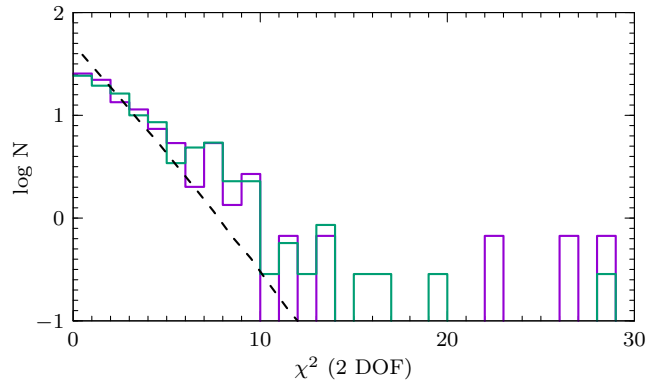


Figure S1. Distributions of χ^2 values for real 149 BL Lacs (magenta histogram) and 350 implants (green histogram). The Gaussian χ^2 distribution is shown by the dashed line. All distributions are equally normalized.

Fig. S1 shows the distribution of offsets in units of χ^2 for individual BL Lacs and implants, i.e. the value $\Delta_x^2/\sigma^2 + \Delta_y^2/\sigma^2$ where $\Delta_{x,y}$ is the offset at each coordinate and σ is the statistical root-mean-square deviation.

One can see, that χ^2 distributions for real BL Lacs and implants are statistically identical. This means that the excessive offsets of GeV images of BL Lacs respectively their optical locations can be entirely prescribed to the inhomogeneous background and a possible contribution of the effect of GeV echo is statistically insignificant.

2. BIAS IN 4FGL CATALOGUE VERSUS OPTICAL COORDINATES

We have found that there is a systematic bias in the locations of *Fermi* catalog objects depending on their optical locations, see Fig. S2. *Fermi* right ascension is systematically higher than that for optical locations for both galactic and extragalactic objects. The difference is up to 0.01° at a high declination. It appears that this is an overall bias that affects the location of each photon. This offset is much smaller than the angular resolution of the LAT, but is critical to our analysis because it is of the same order of magnitude as the offset we are looking for. The bias mimics an EGMF effect, as if most BL Lac jets are inclined in the same direction. This is

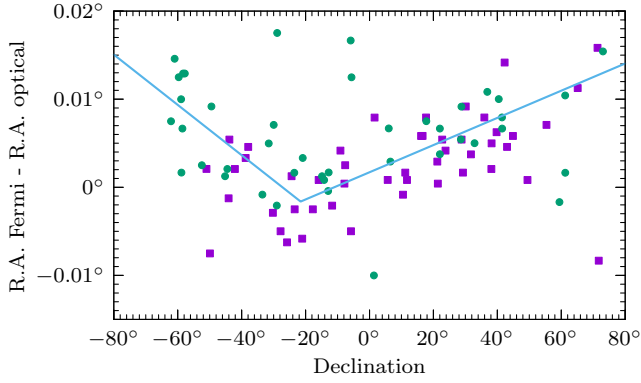


Figure S2. Difference in right ascension for sources in 4FGL catalogue and their known optical coordinates versus declination for 50 brightest extragalactic (squares) and 50 brightest galactic (circles) objects. The broken linear function shows the bias estimate. There is no systematic bias in declination.

unphysical. For this reason, we adjust the coordinates of each photon to account for this bias, estimating it as a broken linear function shown in Fig. S2. The resulting constraint on the EGMF with optical source localization, see Fig. 6 of the main paper, moves it closer to the results obtained using *Fermi* source coordinates. Therefore, the difference between the *Fermi* and optical source locations, excluding statistical uncertainties, can be partially attributed to this bias.

3. APPARENTLY ASYMMETRICAL γ -SOURCES

The absence of statistically significant signal of GeV cascade echo in the subsample of bright objects does not mean that the echo cannot be revealed in some individual weaker blazar images. At the low-luminosity

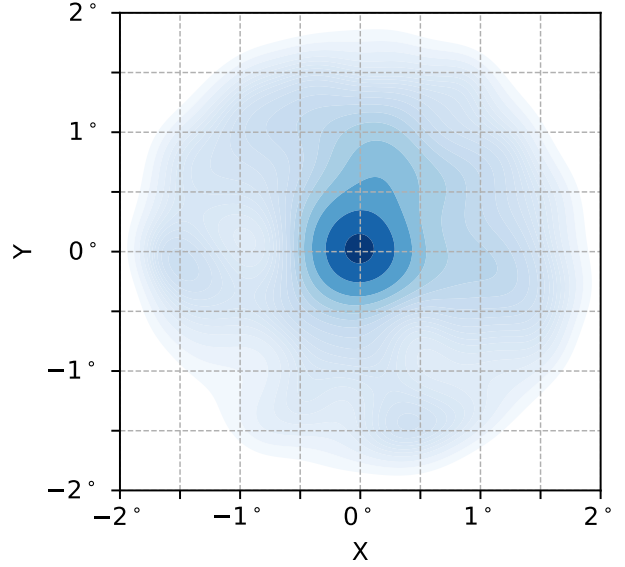


Figure S3. Count map of photons for PKS 0336-177.

end, disregarding the background contribution, the asymmetry can be large if the source is currently in a low-luminosity state and its image is formed predominantly by echoes of its past bright state.

We visually inspected BL Lacs images with large, significant offsets, looking for objects with an extended, asymmetric halo that cannot be easily explained by discrete background sources or a background gradient. The relatively weak PKS 0336-177 is an example of this. The image of this source, Fig. S3, resembles a jet. However, the projected distance would be ~ 5 Mpc which seems too large for a MHD jet. In principle, this can be an effect of a gamma-ray beam from the blazar: photons convert not far from the source, and we observe cascade photons. Objects like this require further investigation.