## Supplementary Material to the article "Perfect absorption by metal-contacted two-dimensional systems with ultra-proximate reflectors"

### I. VARIATIONS IN OPTIMAL DISTANCE TO THE REFLECTOR FOR UNIFORM 2DES WITH COMPLEX CONDUCTIVITY

Deviations of optimal $d$ providing the maximization of absorbance from $\lambda_0/4n_{\rm sub}$ can be observed already at a level of toy model of metal-backed *uniform* 2DES with complex conductivity $\sigma = \sigma' + i\sigma''$ [1]. The Maxwells' equations for the normally incident wave on the layered structure "2DES – substrate – reflector" are solved fully analytically, as the fields throughout all space are uniform (e.g., with transfer matrix technique). The local field in the 2DES plane reads as

$$E = 2E_0 \frac{1 - e^{i\Delta\Phi_g}}{e^{i\Delta\Phi_g}(n - 2\eta - 1) + n + 1 + 2\eta}, \tag{S1}$$

where $\Delta\Phi_g = 2\pi n_{\rm sub} d/\lambda_0$ is the geometric phase shift in the substrate. The absorbance is found from (S1) simply as

$$\alpha = 2\eta' |E/E_0|^2. \tag{S2}$$

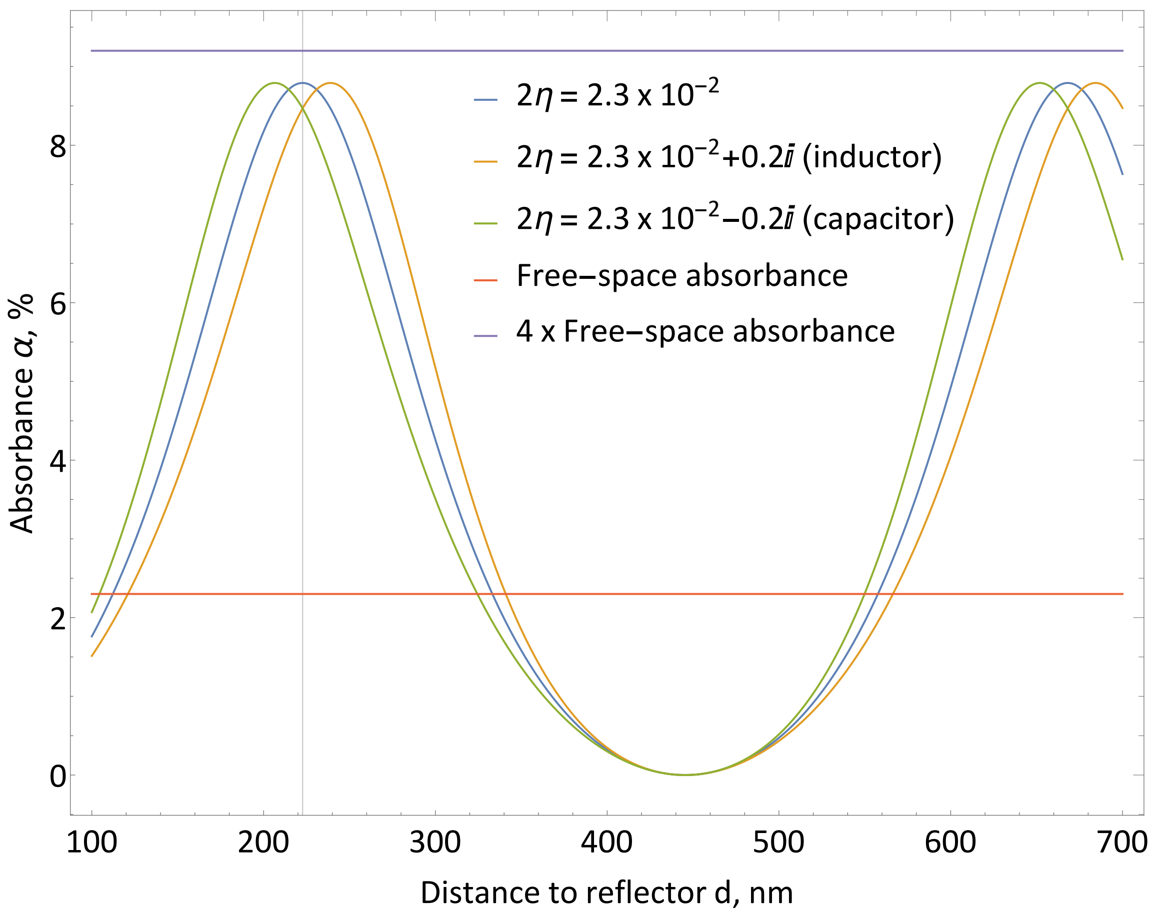


FIG. S1. Absorbance of uniform metal-backed 2DES with real (blue) and artificially added imaginary (orange and green) conductivity vs distance to reflector $d$. The amendment of capacitive (negative imaginary) part of the conductivity shifts the optimal value of $d$ below $\lambda_0/(4n_{\rm sub})$ (marked by vertical gray line). Red and violet lines mark the levels of free-space absorbance 2.3 % and its fourfold multiple 9.2 %. The excitation wavelength is $\lambda_0 = 1550$ nm, the substrate refractive index $n_{\rm sub} = 1.44$

To illustrate the shifts of optimal $d$ with amendment of reactive component of 2DES conductivity, we plot the distance-dependent absorbance computed with (S1, S2) for three representative conductivities $2\eta = 2.3 \times 10^{-2} +$

$\{0, \pm 0.2\}i$, Fig. S1. These correspond to the interband real conductivity of graphene plus some artificial reactive amendment. The excitation wavelength is set to $\lambda_0 = 1550$ nm, the substrate refractive index $n_{\rm sub} = 1.44$.

The optimal absorbance for real conductivity $2\eta = 2.3 \times 10^{-2}$ occurs precisely at $d = \lambda_0/(4n_{\rm sub})$, marked by vertical gray line. With amendment of Im$\eta$, the optimal $d$ shifts from $\lambda_0/(4n_{\rm sub})$. The shift is positive of Im$\eta > 0$ (inductive 2DES) and negative for Im$\eta < 0$ (capacitive 2DES). The demonstrated reduction in optimal $d$ for uniform 2DES with capacitive conductivity provides a simple interpretation why the composite metal-2DES structures (comprising a series of slot capacitors) can be matched at $d < \lambda_0/(4n_{\rm sub})$.

## II. DERIVATION OF OPTIMAL DISTANCE TO THE REFLECTOR FOR METAL-2DES METASURFACE

The optimal distance to the reflector (18) and appearance of the critical value $k_0 L = \pi/n_{\rm sub}$ can be understood already within the lumped model approximation.

In the presence of a back reflector, perfect absorption requires the cancellation of the reactive part of the effective admittance:

$$\operatorname{Im} G_{\rm eff} = \operatorname{Im}(\sigma + G_{\rm rad}) - \omega \operatorname{Re} C_{\rm eff}. \tag{S3}$$

We now consider $f \ll 1$, $k_0 d \ll 1$ and $\eta'' \ll 1$. The radiative admittance contains a singular contribution

$$Z_0 \operatorname{Im} G_{\rm rad} = f n_{\rm sub} \cot(n_{\rm sub} k_0 d) \simeq \frac{f}{k_0 d}. \tag{S4}$$

A singular contribution of the same order is also contained in $C_{\rm eff}$. Keeping only the part enhanced as $1/d$, in the limit $f \ll 1$ we obtain

$$C_{\rm eff} \approx 2f\varepsilon_0 \left[ \frac{L}{2\pi}(1+\varepsilon)\left(\frac{3}{2} - \ln(2\pi f)\right) + \sum_{n=1}^{+\infty} \frac{\varepsilon}{\kappa_\varepsilon^2(q_n) d} \right]. \tag{S5}$$

The sum in the second term in the equation S5 has simple form

$$\sum_{n=1}^{+\infty} \frac{\varepsilon}{\kappa_\varepsilon^2(q_n) d} = \frac{1}{2k_0^2 d}\left(1 - \frac{L}{\lambda_0} n_{\rm sub}\pi \cot\left[\frac{L}{\lambda_0} n_{\rm sub}\pi\right]\right), \tag{S6}$$

which leads to a reduction of the radiation part in the expression for effective admittance

$$Z_0 \operatorname{Im} G_{\rm eff} \simeq \frac{f n_{\rm sub} L}{2d} \cot\left(\frac{\pi n_{\rm sub} L}{\lambda_0}\right) - 2f\frac{L}{\lambda_0}(1+\varepsilon) S_f, \tag{S7}$$

where $S_f = 3/2 - \ln(2\pi f)$. Solving $\operatorname{Im} G_{\rm eff} = 0$ with respect to the reflector distance gives Eq. (18) of the main text.

[1] V. M. Muravev, I. V. Andreev, N. D. Semenov, P. A. Gusikhin, and I. V. Kukushkin, Absorption of electromagnetic waves in a screened two-dimensional electron system, Phys. Rev. B **110**, 205416 (2024).