Supplemental Material to the article "The drop of Hall magnetoresistance as a possible transition to a quantum Hall zero-filling-factor state in a HgTe/CdHgTe double quantum well"

The quality of contacts.

To find out whether the observed effect of the Hall resistivity drop in high fields could be due to a possible contribution of potential contact resistance, we performed the following analysis. The input resistance of the voltmeter is above 10 GOhm. Then, for the observed approximately twofold decrease in Hall resistance to be due to contacts, the contact resistance must increase in the highest fields to approximately the same value as the voltmeter input resistance. Let us compare the measuring circuit containing potential contacts with a current circuit. Both potential and current contacts were made using the same technique. Then, in high magnetic field, the resistance of the current contacts should increase in the same way. The sample current was supplied from a constant voltage source $U = 30$ V and was regulated by the value of the series-connected ballast resistance R_b. To obtain a sample current of 3 μA at zero field, $R_b = 10$ MOhm was set. If, in a high field, the resistance of the current contacts (as well as potential ones) increases to 10 GΩ, this should lead to a drop in current to $30 / 10^{10} = 3\times10^{-9}$ A = 3 nA, i.e. there should be a current drop in the circuit *by 3 orders of magnitude*. In fact, a drop in current in a strong field was indeed observed, but only from 3 to 2.5 μA[1]. As can be easily estimated, such a drop corresponds to an increase in the net resistance of the circuit to 12 MOhm, i.e. to an increase in the resistance of the sample (together with the contacts!) to 2 MOhm, which is 5000 times less than for the option in which the observed feature of the drop in $R_{xy}(B)$ would be explained by an increase in the resistance of the contacts.

In the maximum field of 10.5 T, the measured longitudinal magnetoresistance $\rho_{xx}(B)$ reached approximately $10h/e^2 \approx 260$ kOhm/□. Sample width = 300 μm. Distance between current contacts ≈ 3 mm. Then the resistance over the entire length of the sample should be = 0.26×3/0.3 = 2.6 MOhm. This is comparable to the above estimate of 2 MOhm. Thus we conclude that the resistance of the entire sample in the maximum field is determined mainly by processes in its body (in a two-dimensional volume), but not in the contacts.

Comparison with graphene.

Experimental data, converted into magnetoconductivity versus $\nu = n_s h/eB$ (Fig. S1), contain the plateau $\sigma_{xy}(\nu) = 0$, not as in graphene, but as in traditional semiconductors [1]. This plateau is due to the strong increase in $\rho_{xx}(B)$ in high fields, although the LL pattern contains here a doubly degenerate "zero mode" LL NL = -2 (Fig. 3b), which has some analogy with the zero level in graphene. Probably, the reason for the difference from graphene is connected with the participation of the lateral maximum in conductivity, and the level NL = -2 just passes through it in high fields, that's why, if we place the Fermi level by the gate on the states of this level, then it will be in the region of high density of hole states of the lateral maximum, but this is not the charge neutrality point. In

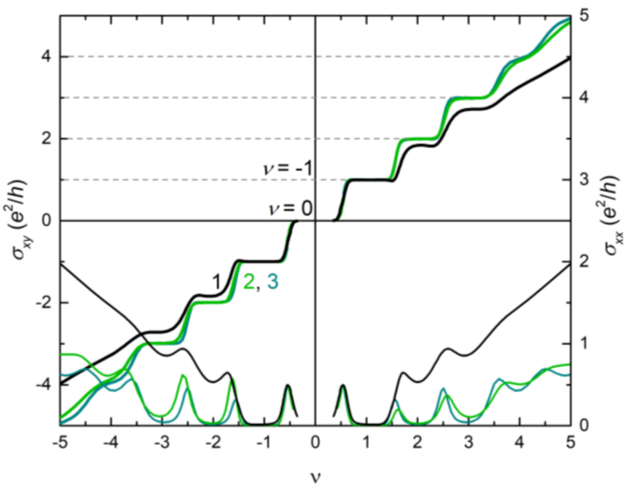

Fig. S1. Magnetoconductivity of Sample 1 before (1) and after illuminations (2, 3). $T = 4.2$ K.

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[1] The drop in Rxy(B) discussed in the article cannot in any way be explained by this drop in current, because at each point in the field, the measured voltage was divided by the actual value of the current.

fact, the charge neutrality point corresponds to the position of the Fermi level inside the gap $\nu = 0$ near the lowest electron LL. It is also possible that this anomalous LL is split due to the built-in perpendicular electric field, since the potential profile of the double quantum well is easily distorted by such a field [2].

Other spectrum options.

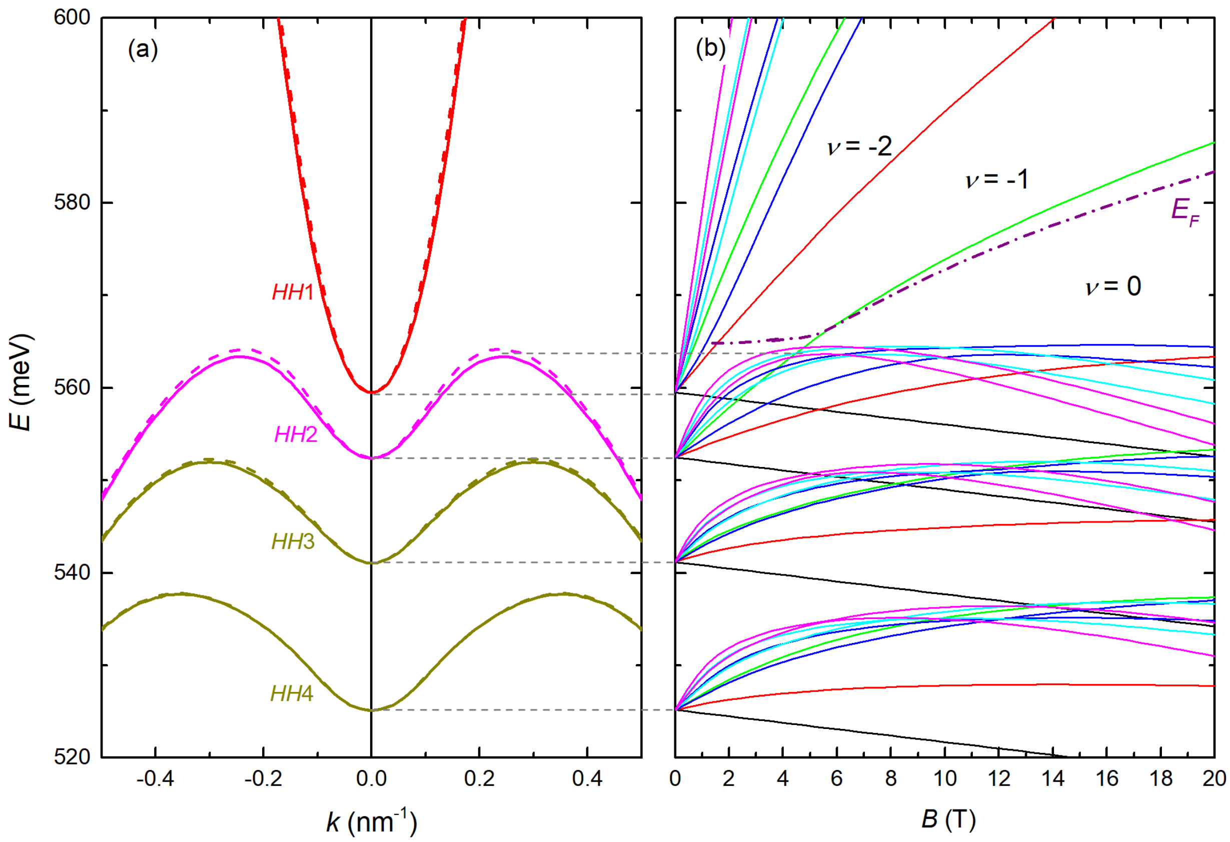

Fig. S2. Single well spectrum as if there was no second layer in Sample 2. In this case, the perpendicular electric field only slightly spin-splits the subbands in the skewed quantum well due to the Rashba effect. In this single-electron approach, the exit of $E_F(B)$ into the gap $\nu = 0$ occurs here in the same way as in the DQW.

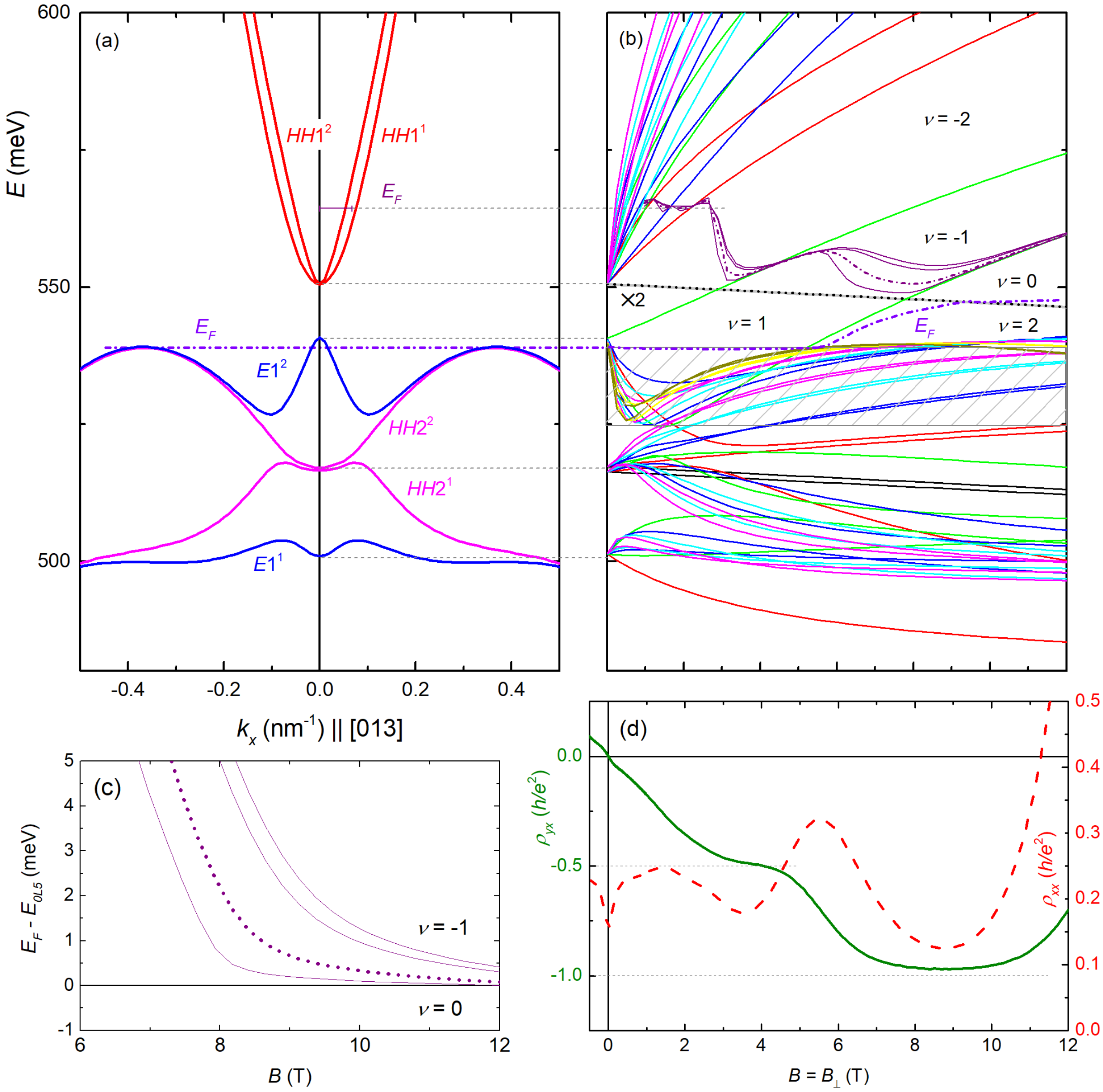

Fig. S3. The band structure (a) and the Landau level pattern (b) of Sample 3. The traces $E_F(B)$ are calculated for the initial state with the light hole density of 0.050×10^{15} m^{-2} (the lower dash-dotted line) and for V_g = +15 V with electron density of 1.35×10^{15} m^{-2} for Γ = 0.5, 1 (dash-dotted), 1.75 and 2 meV. (c) The enlarged fragment of the intersection of $E_F(B)$ with NL = 0 LL. (d) Experimental MR curves for V_g= +15 V to compare with the LL pattern.

1. K. Novoselov, E. McCann, S. V. Morozov, V. I. Fal'ko, M. I. Katsnelson, U. Zeitler, D. Jiang, F. Schedin, and A, K. Geim, "Unconventional quantum Hall effect and Berry's phase of 2*π* in bilayer graphene", Nature Physics, **2**, 177 (2006).
2. M. V. Yakunin, V. Ya. Aleshkin, S. M. Podgornykh, V. N. Neverov, M. R. Popov, N. N. Mikhailov, and S. A. Dvoretsky, "Features of magnetotransport in a HgTe/CdHgTe double quantum well with an intermediate degree of band inversion", JETP Lett. 1**16**, 385 (2022).